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A
PRACTICAL INTRODUCTION
TO
SPHERICS

AND
NAUTICAL ASTRONOMY.

BEING
AN ATTEMPT TO SIMPLIFY THOSE USEFUL
SCIENCES.

CONTAINING,
AMONG OTHER ORIGINAL MATTER, THE DISCOVERY OF A
PROJECTION FOR CLEARING THE LUNAR DISTANCES,
IN ORDER TO FIND THE

LONGITUDE AT SEA;

WITH
A NEW METHOD OF CALCULATING THIS IMPORTANT
PROBLEM.

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La Trigonométrie Spherique est la véritable Science de l'Astrónome.
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P R E F A C E.

ASTRONOMY is allowed to be the most useful as well as the most sublime science that ever engaged the human attention, and the proper foundation of this study is Spherics; for all the heavenly bodies are spherical, or nearly so, and the concave expanse which invests our globe, and in which those bodies appear at equal distances from the eye, is represented by a sphere, upon which circles are drawn, and arcs and angles measured with perfect mathematical precision. Thus the most important problems, both of Astronomy and Navigation, are performed; such as finding the time of the rising, setting, &c. of the heavenly bodies, finding the variation of the compass by azimuths and amplitudes, the latitude by altitudes, and the longitude by the lunar observations.

By the word Sphere is generally understood any orbicular body, but the term was appropriated by the Ancients to an assemblage of circles and constellations

lations representing the Primum Mobile. The invention of this sphere is ascribed to various persons, but it is evidently too remote to be traced by any authentic history. The Chinese * had a knowledge of the Sphere at a very early period, from whom it was probably transmitted to the Chaldeans, and thence into Egypt and Greece†, but was most successfully studied in the famous school of Alexandria. Here Euclid, the celebrated Geometrician, wrote a Treatise on the Sphere, entitled, *Of the Phenomena*, which explained the most interesting parts of ancient Astronomy; such as the Right and Oblique Ascension of the heavenly bodies, with the various other phenomena which arise from the apparent diurnal revolution of the Primum Mobile. This work is supposed the first on the subject perfectly geometrical; it served long after as a model for other performances of the kind, and is still extant, but very scarce.

Hipparchus, who flourished about two centuries after Euclid and one before the Christian æra, is said to have laid the foundation of Spheric Trigonometry. In succeeding ages it was improved by Ptolemy,

* Xuni, 2400 ans avant J. C. fit faire une sphere d'or enrichie de pierres, où l'on voyoit les sept planetes et la terre au milieu. *Martini's History of China*, page 76.

† Thalès qui naquit à Milet 641 ans avant J. C. apporta de l'Égypte dans la Grèce la connoissance des cercles de la sphere, jusques-là ce qu'on avoit entendu par la sphere n'étoit que la description des constellations. *See the late M. Bailly's admirable History of Ancient Astronomy*, page 196.

Theodosius,

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Theodosius, and others; and much is ascribed to Geber, a learned Spaniard, who lived in the sixteenth century; but the most considerable improvements are those of Lord Napier, both by his Proposition of circular parts and his invention of Logarithms.

Within the present century many learned systems have been written upon this subject, chiefly by Messrs. Robertson, Walker, Emerson and Simpson; by La Caille and Mauduit in the French, and by Cagnoli in the Italian; and improvements have been made in the more accurate solutions of certain cases of Spheric Triangles by Dr. Maskelyne, the present Astronomer Royal.

It may seem extraordinary, that while so many great men have contributed to carry the theory to the highest degree of perfection, none have condescended to simplify the practice, or to write a treatise adapted to the common purpose of scholastic instruction; though something of the kind has been so much wanted, that those who taught the science have been mostly obliged to digest manuscripts to teach by. Part of the present work was originally composed for this use, without any view of publication, and the great request which has prevailed for Spherics since its application to the Longitude, induced the author to enlarge and publish his manuscript.

It is not however presumed, that this work should supersede the use, or necessity of those learned systems

already published, being rather intended as an introduction to them, but it will be found particularly useful to persons who cannot devote much time to mathematical investigations, and who chiefly want that part of the science, which is applied in Nautical practice.

A leading object of the present work is to render Stereographic Projection easy and familiar. This subject has been hitherto treated in a style intelligible only to persons conversant in optics; which, though necessary in the theory, may be dispensed with in practice; and even the theory has been objected to, as more scientific than the nature of the subject required; and the late Professor Sanderfon of Cambridge was heard to observe, “that he found the
“greatest difficulty to understand Dr. Halley’s Demonstrations, that the angles upon a sphere were
“equal to their representatives in the Stereographic
“Projection; but when he had laid aside the Demonstrations, and considered the subject in his own
“way, he saw very clearly that it must be so.”*

In the present work the rules of Stereographic Projection are explained in a plain, practical manner, and exemplified by comparing the Figures to the corresponding positions of a globe. In the astronomical part, each problem is first solved upon the globe; the position of which is then represented or taken off

* Dr. Reid’s Enquiry into the Human Mind, p. 124.

by the Projection; a method which has the most sensible and obvious effect in simplifying the subject, though it is believed, it has not been hitherto put into practice.

In order to promote the object of simplification still more, the figures are laid down from the large scale of Gunter, which is supposed to be in the hands of every learner; and as most of the Projections have been measured off on the plates immediately from the scale, it is hoped they will be found as correct as the unavoidable, and sometimes partial shrinking of the paper would permit.

The great use of correct mathematical projection is not perhaps in general sufficiently appreciated. In our universities, projection of any sort is but little attended to, for here the science is said to be studied more with a view to improve the reasoning faculties, than to derive any advantage from practical application; but even in this view it must be owned, that nothing contributes more effectually to fix the attention to the subject than drawing the figures under consideration: when the hands co-operate with the head, the faculties are more collectively, more steadily, and perhaps more agreeably engaged, and the impressions thus made on the memory are the more permanent.

Projection therefore must be highly advantageous even in the theory of pure mathematics, but it is

indispensibly necessary in the practical branches; nor does it appear of greater consequence in any part than in Spherics, where all the circles of the heavens and the earth, with their relative positions and distances, are correctly represented to the eye within the limits of a small piece of paper. Thus a complicated subject is explained by a simple operation, and a difficult task converted to an easy and profitable amusement.

While correctness and simplicity of projection have been here attended to, accuracy of calculation has not been neglected. The problems are all brought out to seconds, a degree of nicety seldom observed in Spherics, though absolutely necessary in astronomical computations.

In the last Section a general view is taken of the longitude, and of the various methods hitherto devised for determining this important problem. The manner of finding the longitude by the lunar observations is explained at some length in an easy, familiar way, and the principles are illustrated by Stereographic Projections, from whence rules are deduced for estimating the correction. As this subject had not been attempted before, it required the more consideration, which has led to the discovery of a method of solving the problem by the projection of four right lines from the plane scale; and though this method cannot be insisted on as perfectly correct, yet, considering the complicated nature of the problem, and the

the great simplicity of the projection, the degree of accuracy must be a matter of surprise rather than of animadversion, as it will be found sufficiently correct for the general purposes of Navigation. Where perfect accuracy is required, this method will be useful as a guide or check to calculation: and it is hoped that the extreme facility of the operation may tend to render the practice of taking lunar distances more frequent among the generality of seamen.

The Book concludes with a new method of working the lunar observations, which has the peculiar advantage of being performed by fines only with one tangent. The various methods hitherto devised for solving this problem display great ingenuity and learning, but they shew at the same time (what might be otherwise demonstrated) the impossibility of doing it by an operation much shorter than that which must take place in the solution of two spheric triangles. Those methods therefore have been chiefly useful as substitutes for Tables calculated to seconds; but these being now provided by the publication of Taylor's Logarithms, the regular method by Trigonometry is certainly preferable to any other: this Dr. Maskelyne seems to allow, by adopting it in his Introduction to the above Tables; and the method here given is founded on the same principles, but so contrived as to avoid the interference of cosines, which greatly assists the memory and prevents mistakes. Nor is the advantage of simplicity its only recommendation: this method is at least equal to any other in conciseness;

ciseness; for it may be observed that some of the others are not in reality so short as they appear: for instance, Dunthorne's, which is generally considered the most concise, requires sometimes a tedious operation in taking out proportional parts from the viiith and ixth of the Requisite Tables; and none of this work appears in the Examples. Another difficulty occurs in the distinction of cases; but in the method here given no distinction of cases obtains, no proportional parts are to be taken, nor can there be any confusion of Tables, or time lost in turning from one to another; and this latter circumstance tends very much to expedite the work; for by the help of a formula (as given on the last plate) and one person being employed to read out the sines, while another sets them down, the operation may be performed in about a third of the time generally necessary where several Tables are to be consulted, and where no such preparation is made; and the solution thus obtained must be perfectly correct, being founded on the unerring principles of Spheric Trigonometry: nor is it even liable to those small inaccuracies which may arise when the answer falls near 90° in the Table of Sines where the logarithmic difference is very small, for here the result comes out the sine of half the true distance, and this cannot be near 90° , as the whole distance is never more than 120° .

Upon the whole, the author's endeavour has been to unite correctness with simplicity, to obviate difficulties

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difficulties hitherto unremoved, and to render a useful but abstruse science more easy and accessible. How far these objects have been effected he now humbly submits to public decision. He only begs to urge in mitigation of errors the great difficulty of attaining perfection in a work, which has some claim to originality both in the plan and execution; a work which, from the variety of new projections and calculations, required much labour and attention, and which has been entirely performed during the spare hours of a laborious profession.

C O N T E N T S.

C O N T E N T S.

THIS work is given in two Parts, each of which is divided into Sections, and numbered with the Roman numerals.

The first Part is subdivided into Articles, which begin with I. and continue regularly to the end. These articles are also marked with the Roman numerals, but somewhat smaller than those of the Sections.

The references which are made throughout the work, and which are inclosed in parentheses, refer to preceding Articles: thus (x) means, see Article the tenth.

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ERRATA

- Page 5, line 14, for alternate read adjacent.]
- 27, 22, for 50' read 50°.
- 38, 22, for TAE read FAE.
- 39, 17, for ABC read ABE.
- 40, 28, for on b read on B.
- 44, 24, for Ar read AB.
- 48, 16, insert Fig. 24.
- 50, 18, for CIX read CXI.
- 64, 14, for Leg read Angle.
- 16, for Adj. Leg AC read Opp. Leg BC.
- 17, for Angle A read Leg BC.
- 17, for Leg AC read Angle A.
- 18, for conjunct read disjunct.
- 23, for BC read A.
- 69, 14, for 9.133, &c. read 9.9133, &c.
- 76, 5, for To Angle read To s. Angle.
- 77, 10 to 14, for As Side, &c. read As s. Side, &c.
- 79, 2, prefix CLXXX.
- 18, for PA read PB.
- 19, for AC read AB.
- 80, 9, for Anology read Analogy.
- 16, prefix CLXXXI.
- 19, for BA read BC.
- 81, 5, for PB read PBC.
- 27, for AB read AC.
- 111, 5, for Apogean read Apogeon.
- 127, 13, for P read E.
- 133, 4, 6 and 9, for 52° read 62°.
- 143, 1, for S⊙T read S*T.
- 162, after 7 insert Zen. dist. $Z\odot = 108^\circ$.
- 165, 21 and 23, and three following Problems, for 26" read 20".
- 178, 7, for 17° 47' 25" read 47° 37' 25".
- 182, 1 and 8, for 2° 55' 26" read 2° 55' 20".

TABLE I. Solutions of Right-angled Spheric Triangles.

Case	Given	Required	Solutions by Analogies.	Refer- ences.	Improved Solutions, logarithmically expressed.
1	Hyp. AB	other Leg AC	$c, BC : R :: c, BA : s, AC$	CLVI	$\left\{ \begin{array}{l} \text{mean } c, BA + 10 - c, BC = c, AC \\ \text{extr. } \frac{t, \frac{1}{2} AB - BC + t, \frac{1}{2} AB + BC}{2} = t, \frac{1}{2} AC \end{array} \right.$
	and a	adjacent Angle B	$R : t, BC :: \cot. AB : c, B$	CLVII	$\left\{ \begin{array}{l} \text{mean } t, BC + \cot. AB - 10 = c, B \\ \text{extr. } \frac{s, AB - BC + 20 - s, AB + BC}{2} = t, \frac{1}{2} B \end{array} \right.$
	Leg BC	opp. $\angle$ A	$s, AB : R :: s, BC^* : s A^*$	CLVIII	$\left\{ \begin{array}{l} \text{extr. } s, BC + 10 - s, AB = s, A \\ \text{mean } t, \frac{\frac{1}{2} AB + BC + \cot. \frac{1}{2} AB - BC}{2} = t, Z \text{ and } 2Z - 90 = A \end{array} \right.$
2	Hyp. AB	other $\angle$ B	$\cot. A : R :: c, AB : \cot. B$	CLIX	$c, AB + 10 - \cot. A = \cot. B$
	and an	adjacent Leg AC	$\cot. AB : R :: c, A : t, AC$	CLXI	$c, A + 10 - \cot. AB = t, AC$
	Angle A	opp. Leg BC	$R : s, BA :: s, A^* : s, BC^*$	CLX	$\left\{ \begin{array}{l} \text{extr. } s, A + s, BA - 10 = s, BC \\ \text{mean } c, AB + 10 - c, AC = c, BC \text{ (AC being known by 5th.)} \end{array} \right.$
3	A Leg AC	other $\angle$ A	$c, AC : R :: c, B :: s, A$	CLXII	$\left\{ \begin{array}{l} \text{extr. } c, B + 10 - c, AC = s, A \\ \text{mean } \cot. \frac{\frac{1}{2} B + AC + \cot. \frac{1}{2} B - AC}{2} = t, Z \text{ and } 2Z - 90 = A \end{array} \right.$
	and its	other Leg BC	$R : t, AC :: \cot. B : s, BC$	CLXIII	$\left\{ \begin{array}{l} \text{extr. } \cot. B + t, AC - 10 = s, BC \\ \text{mean } \frac{s, B + AC + 20 - s, B - AC}{2} = t, Z \text{ and } 2Z - 90 = A \end{array} \right.$



3	and its	8	other Leg BC	A	CLXIII	R : t, AC :: cot. B : s, BC. †	$\left\{ \begin{array}{l} \text{extr. cot. } B + t, AC - 10 = s, BC \\ \text{mean } \frac{s, B + AC + 20 - s, B - AC}{2} = t, Z \text{ and } 2Z - 90^\circ = BC \end{array} \right.$
4	opp. $\angle$ B	9	Hyp. AB	A	CLXIV	s, B : s, AC :: R : s, AB †	$\left\{ \begin{array}{l} \text{extr. } s, AC + 10 - s, B = s, AB \\ \text{mean } \frac{t, \frac{1}{2} B + AC + \text{cot. } \frac{1}{2} B - AC}{2} = t, Z \text{ and } 2Z - 90^\circ = AB \end{array} \right.$
5	A Leg AC	10	other Leg BC	A	CLXV	cot. A* : R :: s, AC : t, BC*	s, AC + 10 - cot. A = t, BC
6	and its	11	other $\angle$ B	A	CLXVI	R : s, A :: c, AC* : c, B*	$\left\{ \begin{array}{l} \text{mean } c, AC + s, A - 10 = c, B \\ \text{extr. } c, A + 10 - c, BC = s, B \text{ (BC being known per 10th.)} \end{array} \right.$
7	adj. $\angle$ A	12	Hyp. AB	A	CLXVII	t, AC : R :: c, A : cot. AB	c, A + 10 - t, AC = cot. AB
8	Both Legs	13	an $\angle$ sup. A	A	CLXIX	t, BC : R :: s, AC* : cot. A*	s, AC + 10 - t, BC = cot. A
9	AC and BC	14	Hyp. AB	A	CLXXVIII	R : c, AC :: c, BC : c, AB	$\left\{ \begin{array}{l} \text{mean } c, BC + c, AC - 10 = c, AB \\ \text{extr. } s, AC + \text{cot. } B - c, B = s, AB \text{ (A being known by 13th.)} \end{array} \right.$
10	Both Angles	15	Hyp. AB	A	CLXX	R : cot. A :: cot. B : c, AB	$\left\{ \begin{array}{l} \text{mean cot. } B + \text{cot. } A - 10 = c, AB \\ \text{extr. } \frac{c, B + A + 20 - c, B - A}{2} = t, \frac{1}{2} AB \end{array} \right.$
11	A and B	16	a Leg, sup. AB	A	CLXXI	s, A : R :: c, B* : c, AC*	$\left\{ \begin{array}{l} \text{mean } c, B + 10 - s, A = c, AC \\ \text{extr. } t, AB + c, A - 10 = t, AC \text{ (AB being known by 15th.)} \end{array} \right.$

The two Quantities marked with Asterisks in the same Analogy are both of the *same Affection*; this mark † means *doubtful*; and in the other Cases the required Quantity is greater or less than 90° , according as the two previous Terms are alike or unlike. (See the References.)

TABLE II. Solutions of Oblique Spheric Triangles.

Cafe and Reference	Given	Analogy.	Required	Solutions by a Perpendicular.	Required	Solutions without a Perpendicular.	References.
1	two Sides	1	< B	$s, AB : s, C :: s, AC : s, B$	< B	$s, AB : s, C :: s, AC : s, B$	
	AB and	2	Side	$\cot. AC : R :: c, C : t, PC$			
	AC	4	BC	$\cot. BA : R :: c, B : t, PB$ $PC + PB = BC$	< A	$s, \frac{1}{2} \overline{AB} - \overline{AC} : s, \frac{1}{2} \overline{AB} + \overline{AC} :: t, \frac{1}{2} \overline{C} - \overline{B} : \cot. \frac{1}{2} A$	CLXXXVIII
	and an opp. $\angle C$	5	Angle A	$s, AC : s, B :: s, BC : s, A$	Side BC	$s, C : s, AB :: s, A : s, BC$	
2	two Sides	6		$\cot. AC : R :: c, C : t, PC$ $BC + PC = BP$			
	AC						
	and	7	Angle B	$s, BP : s, PC :: t, C : t, B$			
	BC	8	Angle A	$s, AC : s, B :: s, BC : s, A$	angles A and B	$s, \frac{1}{2} \overline{AC} + \overline{BC} : s, \frac{1}{2} \overline{BC} - \overline{AC} :: \cot. \frac{1}{2} C : t, \frac{1}{2} A - B$ $c, \frac{1}{2} \overline{BC} + \overline{AC} : c, \frac{1}{2} \overline{BC} - \overline{AC} :: \cot. \frac{1}{2} C : t, \frac{1}{2} A + B$ then $\frac{1}{2} A + B \pm \frac{1}{2} A - B = A$ and B	CLXXXIV CLXXXV
3	incl. $\angle C$	9	Side BA	$s, B : s, AC :: s, C : s, BA$			
		10	Side AC	$s, C : s, AB :: s, B : s, AC$	side AB	$s, A : s, BC :: s, C : s, AB$	
	two < s						
	B and C	12	Side	$\cot. AB : R :: c, B : t, BP$			
CLXXX	and an	13	BC	$t, C : t, B :: s, PB : s, PC$ $PB + PC = BC$	side AC	$s, C : s, AB :: s, B : s, AC$	
	opp. Side AB				$\angle A$	$s, \frac{1}{2} \overline{AB} - \overline{AC} : s, \frac{1}{2} \overline{AB} + \overline{AC} :: t, \frac{1}{2} \overline{C} - \overline{B} : \cot. \frac{1}{2} A$	CLXXXVIII

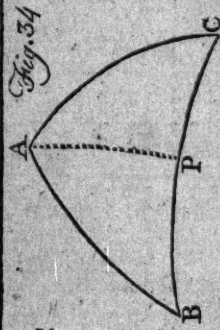


FIG. I.

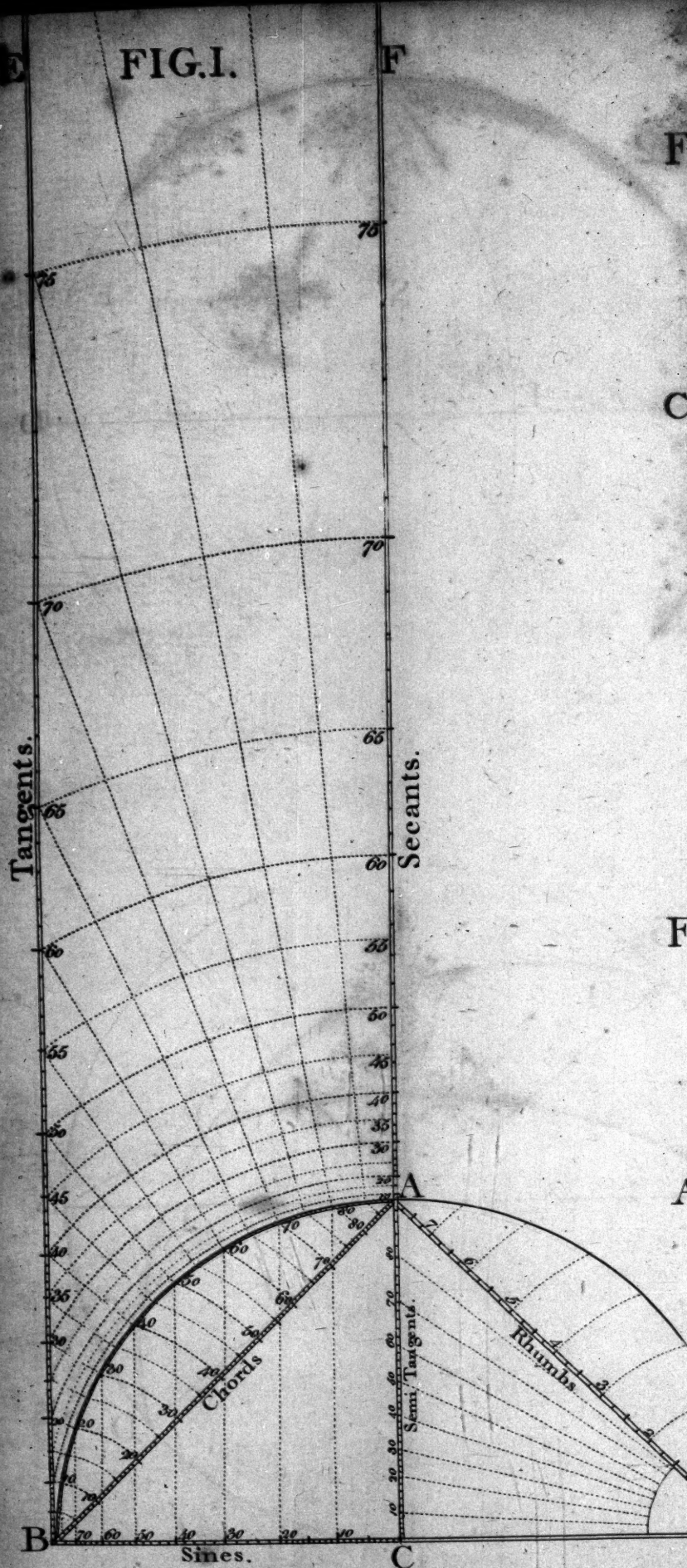


FIG. 2.

The diagram shows a circle with center Z . A vertical line segment AB passes through Z , with A at the top and B at the bottom. A horizontal line segment CD also passes through Z , with C on the left and D on the right. Point S is located on the segment AZ . Point E is on the segment CZ , and point P is on the segment DZ . Point F is in the upper right quadrant, and point y is on the lower right arc. Point n is on the lower left arc. Several lines are drawn: solid lines AF , BF , CF , DF , EF , PF , AF , BF , CF , DF , EF , PF ; dashed lines AS , BS , CS , DS , ES , FS , PS , AS , BS , CS , DS , ES , FS , PS ; and dotted lines AN , BN , CN , DN , EN , FN , PN , AN , BN , CN , DN , EN , FN , PN . Various angles and lengths are labeled: $S.T. 60^\circ$ for CZ , $S.T. 40^\circ$ for EZ , $S.T. 50^\circ$ for DZ , $S.T. 10^\circ$ for ES , $S.T. 60^\circ$ for PF , $Sin. 60^\circ$ for SF , $Cho. 60^\circ$ for CF , $Cho. 40^\circ$ for BE , $Cho. 50^\circ$ for BD , $Cho. 60^\circ$ for AN , $Cho. 40^\circ$ for BN , $Cho. 50^\circ$ for DN , $Cho. 60^\circ$ for FN , $Cho. 40^\circ$ for EN , $Cho. 50^\circ$ for PN , $Cho. 60^\circ$ for AN , $Cho. 40^\circ$ for BN , $Cho. 50^\circ$ for DN , $Cho. 60^\circ$ for FN , $Cho. 40^\circ$ for EN , $Cho. 50^\circ$ for PN .

[illegible]

S P H E R I C S.

SECTION I.

OF THE CHIEF PROPERTIES OF THE SPHERE.

I. **A** SPHERE is a body perfectly round, such as would be formed by the rotation of a circle about its diameter, the diameter being an immoveable axis.

II. Spherics is that part of the mathematics which teaches to measure the arcs of circles described upon the surface of a sphere.

In this science it is indifferent whether the sphere upon which circles are described be great or small, real or imaginary; it is sufficient to suppose a surface perfectly spherical, without any regard either to magnitude or density.

III. The circles of the sphere, that is, the circles described upon a sphere, are distinguished into Great Circles and Lesser Circles.

B

IV. A Great

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S P H E R I C S.

IV. A Great Circle divides the surface of the sphere into two equal parts, as the equator divides the globe of the earth.

V. A Lesser Circle divides the surface of the sphere into two unequal parts, as any parallel of latitude divides the globe.

VI. A Lesser Circle is called a Parallel Circle, referring to some great circle to which it is parallel.

VII. If a sphere were cut by a plane into two equal parts, the section would pass through the centre of the sphere, and the circumference of the section would be a great circle.

VIII. Hence the centre of the sphere is the common centre of all great circles.

IX. It also follows, that no two great circles can be parallel to each other, but will cross in two points diametrically opposite, on the surface of the sphere. Hence as every circle is divided into 360° , the two points of intersection will be 180° asunder.

X. If a sphere were cut by a plane into two unequal parts, the section would not pass through the centre, but the circumference of the section would form a lesser circle.

XI. The great and lesser circles of the sphere are innumerable; for the manner of bisecting a sphere may be varied at every point of its surface; and every great circle thus formed, may be supposed to have an infinite number of lesser circles parallel to it.

XII. The

XII. The nearest distance between any two points on the surface of the sphere, is an arc of a great circle;—for such an arc being described with a greater radius, is less curved than the arc of any lesser circle.

SECTION II.

OF THE POLES OF CIRCLES UPON THE SPHERE.

XIII. EVERY circle of the sphere has two poles diametrically opposite to each other; and a right line drawn through the centre of the sphere from pole to pole, is called the axis.

XIV. The poles of a great circle are each 90° distant from every point of its circumference, as the poles of the world are each 90° from the equator.

XV. The poles of a lesser circle are at unequal distances from the circle, as the poles of the world are with respect to any parallel of latitude.

XVI. No two great circles can have the same poles, (ix) though all have the same centre;—but any number of lesser circles which are parallel have the same poles, though no two can have the same centre.

XVII. Two great circles, crossing at right angles, will pass through each other's poles, and therefore, the pole of a great circle on the sphere may be found by de-

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SPHERICS.

scribing two great circles perpendicular to its plane; and where these circles intersect, will be the pole required.

Thus, to find the poles of the equator—describe any two circles at right angles to it, and they will intersect in the poles. The poles of any lesser circle may be found in the same manner.

SECTION III.

OF SPHERIC ANGLES.

XVIII. A SPHERIC Angle is formed by the inclination of two great circles on the surface of the sphere meeting in a point, called the Angular Point.

XIX. The measure of a spheric angle is an arc of a great circle intercepted between the legs of the angle, 90 degrees distant from the angular point.

Thus, the angle which the equinoctial makes with the ecliptic on the celestial globe, is measured by that arc of the solstitial colure which is intercepted between those circles, and is 90 degrees distant from the angular point.

XX. Hence the angular point is the pole of the great circle which measures the angle;—and the arc of any lesser circle described round this pole, containing the same number of degrees with the great circle, is equally proper to measure the angle, though great circles only are used for this purpose.

XXI. The

XXI. The most practical way to measure a spheric angle, is to find the distance between the poles of the two great circles which form the angle;—for the great circle which measures the angle is perpendicular to those two circles which form it, and therefore passes through the poles (xvii). And as these poles are equi-distant from their respective circles, (xiv) their distance asunder is the measure of the angle.

Thus, the distance between the poles of the ecliptic and equinoctial is the measure of the angle which these two circles make one with the other.

XXII. Two great circles crossing, like two right lines, will make the opposite angles equal to one another, the alternate angles supplements to each other, and the four angles equal to four right angles.

SECTION IV.

OF SPHERIC TRIANGLES.

XXIII. A SPHERIC Triangle is formed by the intersection of three great circles on the surface of the sphere.

XXIV. Both the sides and angles of spheric triangles are computed by degrees, minutes, and seconds, in the same manner as angles only in plane triangles.

XXV. The three angles of a spheric triangle are always more than the three angles of a plane triangle; owing to the convexity of the spheric surface.

XXVI. The three angles of a spheric triangle are always less than six right angles; for one angle cannot be 180 degrees, (xxii) and therefore the three angles must be less than 540°.

Thus, three angles of a spheric triangle may vary from something more than 180 to something less than 540°; and therefore, when two angles are given, the third is not thence known, as in a plane triangle; but when the three angles of a spheric triangle are given, the sides may be thence found;—which cannot be done in any plane triangle.

XXVII. Two sides of a spheric triangle, like those of a plane one, are always greater than the third.

For the sides of a spheric triangle are the nearest distance between the angular points (xii and xxiii) on the sphere, as the sides of a plane triangle are the nearest distance between the angular points upon a plane.

XXVIII. The three sides of a spheric triangle are always less than two semi-circles, or 360 degrees.

For if any two sides be continued, they will intersect in opposite points of the sphere, and form two semi-circles (ix).

Now it is evident that the third side of the triangle, with the produced sides, will make a new triangle;—and as these two new sides are greater than the third, (the
side

side common to both triangles) (xxvii) the two semi-circles must be greater than the three sides of the triangle.

In this spheric triangles differ essentially from plane ones, the sides of the latter being unlimited ;—but in both kinds of triangles the sides may be of the smallest possible dimensions.

XXIX. The following properties are common both to plane and spheric triangles, and may be demonstrated in the same manner.

The greater side subtends the greater angle, and the lesser side the lesser angle ; also, equal angles subtend equal sides, and the contrary.

Two spheric triangles are equal to one another :—1st, When the three sides of the one are equal to the three sides of the other, respectively, that is, each to each.

2^{dly}, When two sides and an included angle of one are respectively equal to two sides and an included angle of the other.

3^{dly}, When two angles and an included side of one are respectively equal to the same of another triangle :—and

4^{thly}, When two sides and their opposite angles, of one triangle are respectively equal to the same of another triangle.

XXX. Two spheric triangles are equal to one another when the three angles of the one are respectively equal to

the three angles of the other;—in this they differ from plane triangles; for the angles of two plane triangles may be respectively equal, and the sides very different.

XXXI. Two sides of a spheric triangle are said to be alike, or of like affection, when both are obtuse, or both acute;—but when one is more than 90° , and the other less, they are said to be unlike, or of different affection.

In the same manner when two angles of a spheric triangle are compared together, they are said to be alike, when both are obtuse, or both acute; or, they are said to be unlike, when one is obtuse and the other acute.

XXXII. If one side of a spheric triangle be produced, the exterior angle is less than the two internal and opposite angles.

For the external and internal adjacent angles are both equal to 180° , (xxii) and the three internal angles are more than two right ones; (xxv) consequently, the external angle is less than the two internal and opposite angles. This is another essential difference between spheric and plane triangles.

XXXIII. Spheric Triangles, like plane ones, are distinguished into equilateral, isoscelar, or scalene, according as three sides are equal, two sides equal, or all three unequal.

XXXIV. Spheric triangles are also distinguished into right-angled, quadrantal, or oblique;—thus, when one of the angles is 90° , it is called right-angled; when a side is equal

equal to 90° , it is called quadrantal; and when neither a side nor angle is 90° , it is called oblique.

XXXV. A Right-Angled Spheric Triangle may have one angle, two, or all three right-angles;—in the latter case it is also quadrantal, and the sides and angles are known.

XXXVI. In a right-angled spheric triangle, like a plane one, the side opposite the right-angle is called the hypotenuse, and the other two the legs; but here there is not that proportion between the hypotenuse and legs as in a right-angled plane triangle.

XXXVII. Every side greater than 90 degrees is greater than the hypotenuse; and every side less than 90 degrees is less than the hypotenuse.

XXXVIII. The hypotenuse is less than a quadrant, if the legs be of the same affection (xxx1); but it is greater than a quadrant if the legs be of different affection.

The hypotenuse is also less or greater than a quadrant, according as the angles at the hypotenuse are of the same or of different affection.

XXXIX. The legs and their opposite angles are always of the same affection.

Other properties of spheric triangles will be given in Spheric Trigonometry.

SECTION

SECTION V.

OF PROJECTIONS.

XL. THE most natural representation of Spheric Triangles is upon a globe, but as figures cannot be conveniently described upon a round body, recourse must be had to projection upon a plane surface.

Projection of the sphere is a representation of all its great and lesser circles, poles, &c. upon a plane surface, such as they would appear to the eye, viewing them from some certain distance, or in some certain position.

In thus projecting the sphere, only one half of its surface is drawn, for the great and lesser circles of both hemispheres are exactly alike: this may be exemplified by setting a globe in any position, and the circles above the horizon will be similar to those below it.

XLI. There are various projections of the sphere which depend upon the two following considerations:—first, the distance or place of the eye;—secondly, the position of the sphere as presented to the eye.

XLII. The distance or place of the eye is distinguished by three kinds of projection, which are called Gnomonic, Orthographic, and Stereographic.

XLIII. Gno-

XLIII. Gnomonic Projection supposes the eye placed at the centre of the sphere, from thence viewing its concave surface. This projection is very useful in some parts of astronomy; but it is particularly adapted to dialling, being the foundation of that science.

XLIV. Orthographic Projection supposes the eye placed at so great a distance from the sphere, as to give it the appearance of a superficies or plane circle; to produce this effect, the eye must be supposed to be at a distance indefinitely great with respect to the diameter of the sphere.

This projection is useful in some parts of astronomy; particularly, in determining the transits of planets over the sun's disk, and in the projection of lunar and solar eclipses.

XLV. Stereographic Projection, (which shall be more particularly explained in the next section) supposes the eye placed at some point of the surface of the sphere.

This projection not only affords the most natural representation of the prominence and convexity of the sphere, but is also performed with more ease and accuracy than any other; for in the gnomonic and orthographic projections, the circles of the sphere must be sometimes represented by elliptic and hyperbolic curves, which are difficult to be described accurately; whereas in stereographic projection, all the circles of the sphere are represented either by circles or by right lines. The stereographic method is therefore justly preferred to any other, both for its simplicity and correctness; and is the only kind of projection used in this work.

XLVI. That

XLVI. That variety of projections which arises from the second consideration, that is, the position of the sphere, as presented to the eye, (xli) equally occurs in any of the three foregoing methods: some specimens are given in the astronomical part of this work, with general rules for all such problems.

There are, beside the foregoing, various other projections of the sphere, which do not immediately apply to Spherics, such as the Globical and Cylindrical projections; the Scenographic, which belongs to perspective, and the Mercator's, which is applied in navigation; the latter is sometimes used in geography, but the common maps of the world, and of countries, are all drawn upon stereographic principles. Stereographic Projection is therefore necessary toward a proper knowledge of geography; but a globe only can give the true proportional dimensions of countries.

For in the maps of the world, the dimensions must be gradually contracted from the out-line to the centre, and therefore those countries alone, which lie on the border of the map, are truly represented; this gradual contraction however can be allowed for, and distances correctly measured; but these things can only be performed by persons who understand stereographic projection.

SECTION

SECTION VI.

OF STEREOGRAPHIC PROJECTION.

XLVII. The term *Stereographic* is derived from two Greek words, which signify *to describe a Solid*. In Spherics it means a linear representation of the sphere upon a plane, such as it would appear to the eye viewing it from some point of its surface.

XLVIII. The eye thus placed is supposed to see one half of the convex surface of the sphere; the circle therefore which terminates the view, is a great circle, (IV) and the place of the eye is the pole of this circle (xiv).

XLIX. In projecting the sphere, that great circle which terminates the view is first made, and is called the Primitive Circle; the place of the eye, (its pole or centre) is called the Projecting Point, and the plane upon which the projection is made, is called the Plane of Projection.

L. It is easy to conceive, that in such a projection, the convex surface of the hemisphere must be contracted to a plane surface of the same diameter as the sphere, and therefore the primitive circle only will retain the primitive radius of the sphere; the inner great circles will appear gradually distended or straightened from the primitive to the projecting point; thus, the great circle next the primitive will be of less curvature, that is, described with a greater radius than the primitive, and
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this effect gradually increases to the centre, or projecting point, insomuch that any great circle passing through this point loses all curvature; for being seen edgewise, it appears as a right-line, and must be projected accordingly.

LI. These various appearances of the great circles in the projection, may be exemplified by viewing a thin ring or coin in any of the above positions: thus, when viewed horizontally, the edge will appear under its proper size and form; viewed obliquely, the curve will seem straightened, and if held edgewise to the eye, it will appear as a right-line, or diameter of the circle.—Or

If an armillary sphere were cut by a plane into two equal parts, and one of the sections set upon a table with a light at the vertical point, the shadow of this hemisphere would exhibit a true specimen of Stereographic projection.

For by the laws of shadow in perspective, the effects of light and vision are similar, and therefore those circles of the armillary sphere, would appear under the same form in the shadow, as they would to an eye, in the place of the light.

If the section be supposed to pass through both poles, it is easy to conceive, that the curvature of the great circles in the shadow will gradually decrease from the outline to the centre; and that any great circle passing immediately under the light, will cast a shadow like a right line.

SECTION

SECTION VII.

OF THE GREAT CIRCLES OF THE SPHERE IN THE
PROJECTION.

LII. FROM what has been said, it is evident that there can be but three varieties of great circles in the projection; these are called the Primitive Circle, a Right Circle, and an Oblique Circle.

LIII. The Primitive Circle is the outline first made, which encompasses the projection, as A C B D.—(Fig. 2.)

LIV. A right circle being a great circle seen edgewise, is always a diameter to and at right angles with the primitive, as A B or C D.

LV. An oblique circle is any great circle between a right one and the primitive, as A E B; it always cuts the primitive in opposite points, as A and B (x); and therefore, a right line drawn through the extreme points of an oblique circle, will pass through the projecting point, as A B passes through Z.

LVI. The radius of an oblique circle is always greater than that of the primitive;—and it gradually increases as the oblique circle approaches toward the projecting point.

A right line crossing the oblique circle at right angles, and produced to any necessary length, is called the Line
of

of Measures;—thus, if x be the centre of AEB — Ex is the line of measures.

Any line upon which the centres of lesser circles fall, is also called the Line of Measures. The properties of lesser circles in the projection will be explained in the problems.

SECTION VIII.

OF THE POLES OF GREAT CIRCLES IN THE PROJECTION.

LVII. THE poles of any great circle in the projection are 90 degrees distant from the periphery of that circle (xiv).

LVIII. The pole of the primitive circle is therefore the centre, or projecting point (xlviii);—And,

The farther pole of this circle is supposed to be in the opposite point of the lower hemisphere.

LIX. The poles of a right circle are in the primitive 90 degrees from each end of the right circle; thus, A and B are the poles of the right circle CD , and the points C and D , are the poles of AB .

LX. The pole of an oblique circle is in the line of measures 90 degrees from the circle; thus, p is the pole of
of

A E B, for $Z E = 40^\circ$, and $Z p = 50^\circ$, (which shall be shown in article, LXXIII.)

LXI. The other pole of the oblique circle is supposed on the opposite point the lower hemisphere;—it may be found in the projection by producing lines beyond the primitive, and is then called the Exterior Pole.

SECTION IX.

FIG. 1.

OF THE CONSTRUCTION AND USE OF THOSE LINES UPON GUNTER'S SCALE, WHICH ARE APPLIED IN STEREOGRAPHIC PROJECTION.

THE Lines of Gunter's Scale, which are used in stereographic projection are the chords, secants, tangents, and semi-tangents; they begin at the middle of the scale, and are marked—Cho. Sec. Tan. and S*T.

OF THE LINE OF CHORDS.

LXII. Def. The Chord of an Arc is a right-line drawn from one end of the arc to the other, thus the right-line B A is the chord of the arc B A.

LXIII. Constr. Let the arc B A be divided into 90 equal parts or degrees;—mark the tenth divisions with their respective numbers, 10° , 20° , 30° , &c. to 90° . Then with one leg of the compasses on B, as a centre, transfer those several divisions of the arc to the chord B A, and this marked with the corresponding figures of the arc will be a line of chords.

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LXIV. Use.

LXIV. Use. The line of chords is used to project the primitive circle, and any arc of the primitive is therefore measured by the same scale.

Thus, the primitive, $A C B D$, Fig. 2, is described with the chord of 60 degrees, and any arc of this circle is measured on the line of chords in the same manner as an angle of a plane triangle.

OF THE LINE OF TANGENTS.—FIG. 1.

LXV. Def. The Tangent of an Arc is a right-line touching that arc at one end, and is always terminated by a secant drawn through the other end.

LXVI. Constr.—The arc $B A$ being divided as before, (LXIII) a ruler laid on C , and these several divisions of the arc will give their corresponding divisions, in the line $B E$, which is thus made a Line of Tangents, and is to be accordingly marked from B to E with 10, 20, 30, &c.

LXVII. Use.—The Line of Tangents is used to find the centres of oblique circles in the line of measures.

Thus, if it were required to project the oblique circle, $A E B$, (Fig. 2) to make a given angle with the primitive, the tangent of this angle will extend from Z the projecting point, to x in the line of measures, and x is the centre of the oblique circle, $A E B$.

The line of tangents is also used to describe lesser circles, but this with its other uses shall be shewn in the Problems.

Or

OF THE LINE OF SECANTS.—FIG. 1.

LXVIII. Def.—The Secant of an Arc is a right-line drawn from the centre, through one end of the arc, and terminated by the tangent of the same arc: thus, a ruler on C, and the several divisions on the arc B A, will coincide on the line of tangents with the corresponding figures of those divisions, and the distances from C to these figures, will be the secants of these arcs respectively.

LXIX. Constr.—The Line of Secants, C F, is constructed by taking the distances from the centre C, to the divisions on the line of tangents, and with one leg of the compasses on C, transferring those distances to the line C F, which will give the respective divisions of the line of secants.

These divisions are numbered from A towards F, for as the secant of the smallest angle is more than radius, this distance (viz. C A) must be allowed before the line of secants begins,—upon the scale, this space is occupied by the line of fines.

LXX. Use.—The line of secants is applied to the projection of oblique circles: thus, if it be required to draw an oblique circle, (A E B) to make a given angle (C A E) with the primitive, the secant of the required angle is the radius of the oblique circle; if therefore this secant be taken in the compasses, with one leg upon A or B, it will find x in the line of measures, and x is the centre of the required oblique circle.

Other uses of this scale will be shewn in the Problems.

OF THE LINE OF SEMI-TANGENTS.—FIG. 1.

LXXI. Def.—The Semi-tangent of an Arc means the tangent of half that arc : thus, C A is a line of semi-tangents, and is equal to the tangent of 45° ; that is, equal to the tangent of half the quadrantal arc B A.

LXXII. Constr.—A ruler on D and the several divisions of the arc B A, will cut the radius C A in the corresponding divisions of semi-tangents, which are marked accordingly.

Here it is evident, that the semi-tangent of any arc (suppose 40°) is equal to the tangent of (20°), half that arc; for by the 20th Proposition of the Third Book of Euclid, an angle at (C) the centre, is double an angle (at D) at the periphery of a circle.

The line of semi-tangents is continued upon the Gunter, as far as the length of the scale will admit: these divisions, beyond 90 degrees, may be found by dividing the arc D A like the arc B A, and a ruler laid on D and these several divisions, would give the required semi-tangents upon the line A F.

LXXIII. Use.—The line of semi-tangents is applied to the measurement of right circles: thus, the arc Z E is measured by this scale, and is 40 degrees.—Fig. 2.

LXXIV. If

LXXIV. If the measure of an arc of a right-circle be taken from the periphery, toward the centre, it is reckoned backward on the line of semi-tangents: thus, the arc C E will extend on this scale from 90 degrees downward to 40 degrees, and is therefore 50 degrees.

LXXV. The lines of fines and rhumbs have no necessary connection with the subject of stereographic projection, but they are placed here to fill up those vacant spaces in the figure.

The line of fines C B is constructed by drawing lines parallel to the radius C A, through the several divisions of the arc B A, and the line of rhumbs is made by dividing the arc A D into eight equal parts, and with one leg of the compasses upon D, transferring each division to the chord D A, which will be the line of rhumbs.

SECTION X.

FIG. 2.

THE USE OF THE SCALE FURTHER EXPLAINED AND ILLUSTRATED.

LXXVI. IT has been shewn in the foregoing section, that the primitive circle is projected and measured by the line of chords, a right-circle by the line of semi-tangents, and that an oblique circle may be projected either by the line of tangents or of secants; these properties will be

better understood by the following explanation of the diagram.—Fig. 2.

LXXVII. Here let it be required to draw an oblique circle, A E B, to make a given angle, suppose 50 degrees, with the primitive.

First, with the sweep of 60 degrees from the line of chords, describe the primitive circle A C B D, draw the right-circles A B and C D at right-angles to each other, and let either be produced as a line of measures:—suppose Cx.

LXXVIII. Then take the tangent, 50 degrees, the required angle, and with one leg of the compasses on Z, the other leg will extend to x, the centre in the line of measure.

Or, take the secant 50 degrees, and it will extend from A or B to x; for the secant of the angle is always the radius of the oblique circle, (LXVII.)

LXXIX. Now if a plane angle, Z A F, be laid off equal to 50 degrees, and the line A F continued, it will meet the line Z D (produced) in x.

For Z x being the tangent 50 degrees, and Ax its secant, (LXXVIII) it is obvious that Z F must be the chord, and S F the sine of the same angle.

LXXX. Again, let the angle Z A F be divided into two equal parts by the line A p, and it is plain that Z p is the tangent of half 50 degrees; or, otherwise expressed, it is the semi-tangent of 50 degrees.

Let

Let $A p$ be continued to y , and let $Z y$ be drawn.

Then by the 20th Prop. of the third of Euclid, the angle $(B Z y)$ at the centre, is double the angle $(B A y)$ at the periphery, and therefore the arc By , is equal to the chord of 50 degrees.

LXXXI. Hence the arc $(Z p)$ of a right circle, contains the same number of degrees upon the scale of semi-tangents, that its corresponding arc $(y B)$ of the primitive contains upon the scale of chords.

Thus any arc of a right circle may be measured by drawing two lines from the pole of the circle to intercept the arc, and these continued to the primitive will there give the measure required.

LXXXII. In the same manner, any arc of an Oblique Circle is measured by drawing two lines from its pole to intercept the arc, and these continued to the primitive will there give the required measure.

For all right and oblique circles may be considered as primitives, contracted (L), and therefore measuring them by their corresponding arcs upon the primitive, is as it were restoring them to their original magnitude.

SECTION XI.

OF THE CONSTRUCTION AND USE OF THOSE LINES
UPON THE SECTOR WHICH ARE APPLIED IN STEREOGRAPHIC PROJECTION.

LXXXIII. THE lines upon the sector which are applied in stereographic projection, are the same as those upon the plane scale—namely, Cho. Tan. and Sec. they are constructed upon the same principles, and may be used in the same manner.

The difference then between these two instruments consists in the same lines being entered doubly upon the sector; viz. once upon each leg, by which they may be used two ways, namely, laterally, and transversely.

LXXXIV. *Lateral Distance*, is that taken lengthwise, or according to the line, (like the plane scale) beginning at the centre of the joint or axis of the sector.

LXXXV. *Transverse Distance*, is that taken crosswise, between any two corresponding divisions of the scales of the same name, the sector being opened in an angular position. The scales of this instrument being marked with two or three parallel lines, the transverse distances are taken with the points of the compasses set on the lines next the inner edge of each leg, for these lines only, run to the centre of the sector.

LXXXVI. The

LXXXVI. The proportion between lateral and transverse distances, arises from the nature of similar triangles; for, let the sector be set in an angular position, and it is evident that any number of lateral distances, with their transverse distances respectively, form a series of similar triangles; and therefore, as any lateral is to its transverse, so is any other lateral to its transverse.

Thus, suppose upon the line of equal parts, called the line of lines, marked lin: If the lateral 2, be taken in the compasses, and the sector opened until this distance extends across between the nearest points of 4; all other transverse and lateral distances respectively, will be to each other as two to one; that is, as the lateral two, is to the transverse four, so is the lateral three to the transverse six; and so of any other number.

THE USE OF THE CHORDS ON THE SECTOR.

LXXXVII. The advantage of using the scale of chords transversely, consists in being able to vary the size of the projection to any radius within the limits of the sector. Thus, the legs may be opened or closed to any proposed angle, making the transverse distance between the points 60° and 60° of the scale of chords equal to the radius proposed; and any angle may be measured transversely, from 60 downward to the centre of the sector.

LXXXVIII. When the angle which is to be measured or projected exceeds 60° , (the limits of the sector,) it is to be performed by taking the angle in two chords or
three

three. Thus, if it be required to lay off an angle of 100° degrees.

With any opening of the sector, take the sweep of 60° , which is the transverse distance between the extreme points of the scale of chords.

Then lay off two chords, which will make both 100° ; suppose 40° and 60° , or any other two or three chords under 60° , which together make 100° , and they give the angle required.

To describe the primitive in Fig. 2, and measure an arc thereon, open the sector till the transverse distance between 60° and 60° on the chords equals ZA ; with this radius, describe the primitive $ACBD$, and any arc thereon is measured transversely between the same lines of chords.

THE USE OF THE TANGENTS ON THE SECTOR.

LXXXIX. These scales may be used transversely, to find the centres of oblique circles, or to measure the arcs of right circles, like the line semi-tangents upon the plane scale.

There are two lines of Tangents on each leg; the first begins at the centre, and continues to 45° to the end of the sector, these are called the lower tangents; the second begins at 45° , one fourth part of the length of the sector from its centre, and continues to 75° , and these are called the upper tangents.

To

To find the centre of an oblique circle, which makes a given angle with the primitive.

XC. If the given angle be under 45° , set the sector as before (LXXXVII), that is, let the radius of the primitive equal the transverse distance between the points 45° on the lower tangents; and any transverse of these lines, under 45° , laid off from the centre of the primitive on the line of measures, will give the centre of the oblique circle required.

XCI. If the given angle be more than 45° , then the operation is to be upon the upper tangents.

Thus, in Fig. 2, if it be required to describe an oblique circle, A E B, to make the angle C A E equal 50° .

Take the radius of the primitive in the compasses, and open the sector until this distance will extend across from those two points of 45° , which begin the upper tangents.

Then take the transverse 50° from the same scales, and this laid off from Z, will give x the centre required.

XCII. Any arc of a right circle is measured by the transverse of the tangents, by opening the sector till the transverse 45° and $45'$ on the lower tangents equals the radius of the primitive.—As in Art. xc.

Thus, to measure the arc Z E of the right circle, the sector being set as above, this arc will equal the tangent 20° , which is the half-tang. 40° .

When

When the given arc begins at the primitive, it is measured by finding its complement to the centre: thus, CE is measured by finding ZE, as in the last case, and subtracting it from 90° .

If the arc be more than 45 degrees proceed as in article XCI.

THE USE OF THE SECANTS UPON THE SECTOR.

XCIII. The secants of the sector may be used to find the centre of any oblique circle:—this is performed by opening the instrument till the radius of the primitive circle extends transversely between those points where the secants begin to be numbered, which are each $\frac{1}{4}$ part of the length of the sector from its centre.

Thus, the sector is used with the same opening for the secants as for the upper tangents.

XCIV. If it be required to find the centre of an oblique circle, which should make an angle 50 with the primitive, as AEB in Fig. 2.

Describe the primitive with a radius equal to the transverse between the points where the secants begin (xciv); then the transverse 50 laid off from A or B, will give x in the line of measures, which is the centre required, and this secant is therefore the radius of the oblique circle AEB.

There are besides the above lines on the sector, scales of logarithmic sines, tangents, and numbers, (as upon the Gunter;) these are used with the instrument quite open, as they extend upon both legs. The other lines of the

the scale and sector are chiefly useful in navigation, and are described in most books upon that subject.

The sector is an instrument of extensive use in most branches of the mathematics; its excellence in stereographic projection is found where the size of the figure should be varied (LXXXVII); but where no such variation of radius is necessary, the plane scale is perhaps preferable: for lateral distances are taken with more ease and expedition than transverse distances; besides, the given radius of the sector is liable to be moved from its position; whereas, the radius, or chord of 60° , upon the Gunter, is unchangeable.

What has been hitherto delivered being well understood, the learner may proceed to the Problems:—Each of which should have its appropriate figure, and the dimensions may be varied at pleasure; and nothing will tend more to illustrate the subject than to set a globe in a position to correspond with each figure.

Thus, to set a globe to correspond with Fig. 2:—

Elevate either pole 40 degrees above the horizon; then will this pole represent the pole p; the equator will represent AEB; the brazen meridian will represent CD; the vertex of the globe will represent Z; and the circle which coincides with the wooden horizon will represent the primitive ACBD.

The Problems being partly an application of the foregoing sections, repetitions are unavoidable.

SECTION

SECTION XII.

STEREOGRAPHIC PROBLEMS.

PROBLEM I.

XCV. *Through any three given points, to describe a circle:*—Bisect the distance between any two of those points, and the intersection of the two bisecting lines will be the centre required. (Euclid p. 25, b. 3.)

PROBLEM II.—FIG. 2.

XCVI. *Through any given point within the primitive circle, to describe a great circle:* this admits of two cases—

1. *If the required circle should be a right one:*—Draw a right line through the given point and the centre of the primitive.

2. *If the required circle should be an oblique one:*—Draw a diameter near the given point, and through both ends of the diameter and this point describe a circle (xcv:)

Thus, if the given point be E, draw a diameter, as AB; then through the three points A E and B describe the circle A E B.

PROBLEM

PROBLEM III.—FIG. 2.

XCVII. *To find the pole of any great circle:—*The great circles in the projection being of three kinds; this problem admits of three cases.

XCVIII. Case 1. *To find the pole of the primitive circle:—*The centre and pole of the primitive are the same, (LVIII.)

Case 2. *To find the pole of a right circle, suppose A B:—*

XCIX. Take 90 degrees in the compasses, and from either end, A or B; it will extend to C or D in the primitive, which are the poles required (LIX.)

Case 3. *To find the pole of an oblique circle, suppose A E B.*

C. Draw a right line from the angular point A, through the point E, which continued will give n in the primitive.

From the scale of chords lay off 90 degrees from n to y.

Draw y A, which will cut C D, the line of measures in p. Then is p the pole of the oblique circle, A E B: or,

Measure the distance Z E, and lay off its complement the other way from Z, which will extend to p (LXXIII), the pole required:—or,

If the angle which the oblique circle makes with the primitive be known, (suppose $\angle CAE = 50^\circ$) then the semi-tangent of this angle (50°), laid off from Z, will extend to p, the pole required.

PROBLEM IV.—FIG. 2.

TO MEASURE ANY ARC OF A GREAT CIRCLE.

This problem also admits of three cases.

Case 1. *To measure any arc of the primitive.*

CI. Any arc of this circle is measured upon the scale of chords, thus $Cn = 50$ upon this scale.

Case 2. *To measure an arc of a right circle.*

CII. An arc of a right circle is measured by the scale of semi-tangents (LXXIII and LXXIV), or by its corresponding arc of the primitive (LXXXI); thus, the semi-tangents $ZE =$ the cord nB .

Case 3. *To measure any arc of an oblique circle.*

CIII. Any arc of an oblique circle, is measured by drawing two lines from its poles (LXXXII) to intercept the arc, and these continued to the primitive, will there give the required measure on the line of chords.

Thus,

Thus, the sides in the spheric triangle CAE , in Fig. 3, are measured.

The pole of AE is found in q ; (c) then a ruler on q and E will give e in the primitive; and on q and A gives A ; hence the chord AE is the measure of the arc or side AE , and $= 145^\circ$.

In the same manner the pole p of CE being found, (c) a ruler on p and E will give f in the primitive; and on p and C gives C : hence the chord Cf is the measure of the arc or side CE , and $= 116^\circ$.

The remaining side AC is measured upon the chords, and equals 40° .

PROBLEM V.—FIG. 2.

TO MEASURE ANY SPHERIC ANGLE.

This problem admits of three cases:—

Case 1. When the spheric angle is at the centre, as BZy .

CIV. The chord yB is the measure required.

Case 2. When the spheric angle is at the primitive, as, CAE .

CV. The arc CE is the measure of this angle, (LXXIV) and will be found $= 50$ degrees; its corresponding arc Cn is also the measure, (LXXXI.)

D

For

For the measure of a spheric angle is an arc of a great circle intercepted 90 degrees from the angular point, (xix) and C E is 90 degrees from A.

CVI. An angle at the primitive as C A E, may be also measured by finding the distance between the poles of the two great circles which form the angle (xxi); this being a general rule for measuring all kinds of spheric angles.

Thus, a ruler on the angular point A and the pole p of the oblique circle will give y in the primitive; and on A and the pole Z of the primitive will give B in the primitive; hence the chord y B, is the measure of the angle C A E.

CVII. That the arcs C E and Z p, are equal to one another, may be thus proved: the arc C Z = 90 degrees (a quadrant), and the arc E p = 90, (the distance of the oblique circle from its pole).

Hence C Z = E p; and if the arc E Z which is common to both quadrants be taken away, the remaining arcs C E and Z p will be equal to one another.

Case 3. When the spheric angle is neither at the centre nor at the primitive, as C E A in Fig. 3.

CVIII. Find the poles p and q of the two great circles C E D and A E B which form the angle. (c.)

Then a ruler on the angular point E, and those two poles p and q will give x and y in the primitive; and the chord x y is the measure of the angle E, (cvi.)

In

In the same manner the angle BFD in the small triangle in Fig. 3 is found.

This angle is formed by the oblique circle CED , whose pole is p ; and by the right-circle AB , whose pole is L .

A ruler therefore on the angular point F , and the poles p and L will give the required measure on the primitive.

PROBLEM VI.—FIG. 2.

TO DESCRIBE AN OBLIQUE CIRCLE TO MAKE A GIVEN ANGLE WITH THE PRIMITIVE.

This useful problem may be performed by the chords, secants, tangents, or semi-tangents.

CIX. Thus, suppose it were required to describe the oblique circle AEB , to make an angle of 50° with the primitive.

1. The angle ZAF being laid off from the chords $= 50^\circ$, and AF continued, it will meet ZD produced in x :—then is x the centre of the oblique circle ($LXXIX$.) Or,

2. The secant 50° is the radius of the required oblique circle, and therefore will extend from A or B to x , the required centre (LXX .) Or,

3. The tangent, 50° degrees, will extend from Z the projecting point, to x , the centre of the required circle, ($LXVII$.) Or,

D 2

4. The

4. The semi-tangent 50 degrees laid off from C will extend to E, (LXXIV), then through the three points A E and B describe a circle, (xcv.)

The second method is the most practical, being performed by one opening of the compasses.

To shorten the directions in the following constructions, it is to be understood, that the primitive is always first described with two diameters at right-angles to each other.

It should also be understood, that the small letters marked on the plates in ITALICS, are given here in the ROMAN CHARACTERS:—thus, x or y on the plates, are represented by x and y in the print.

FIG. 1

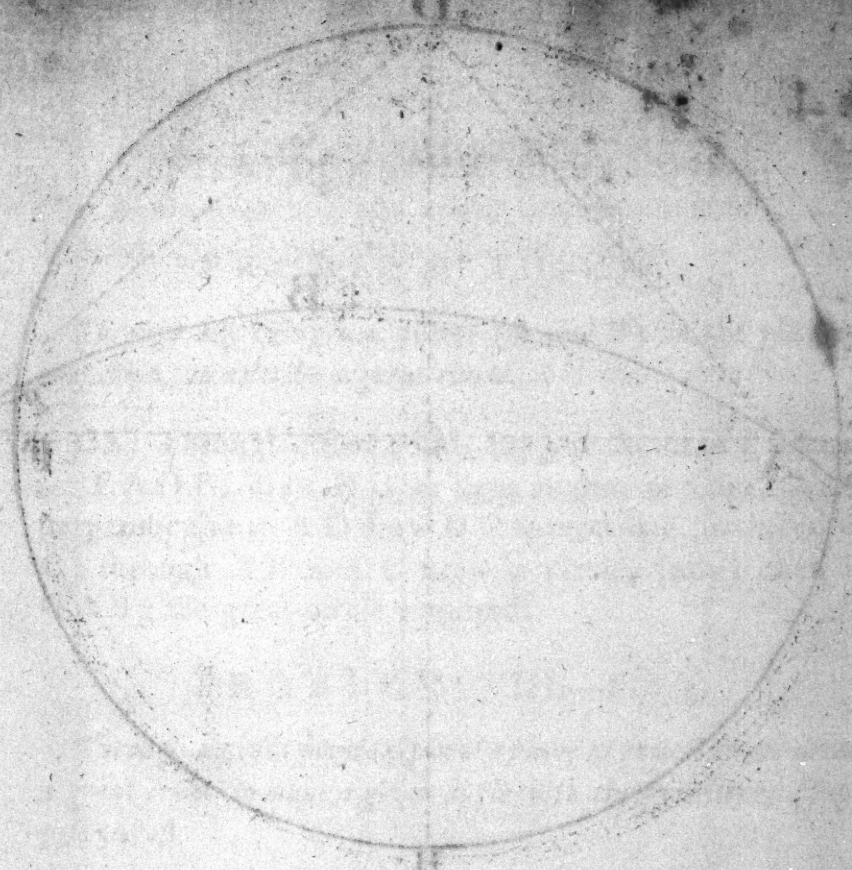


FIG. 2

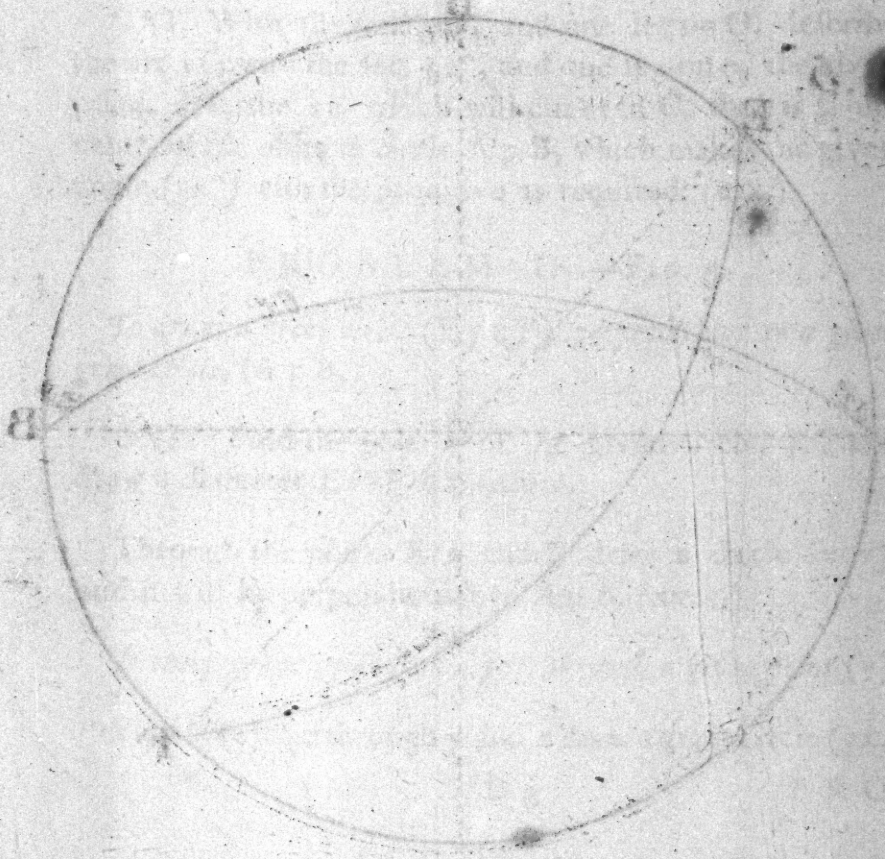


FIG. 4.

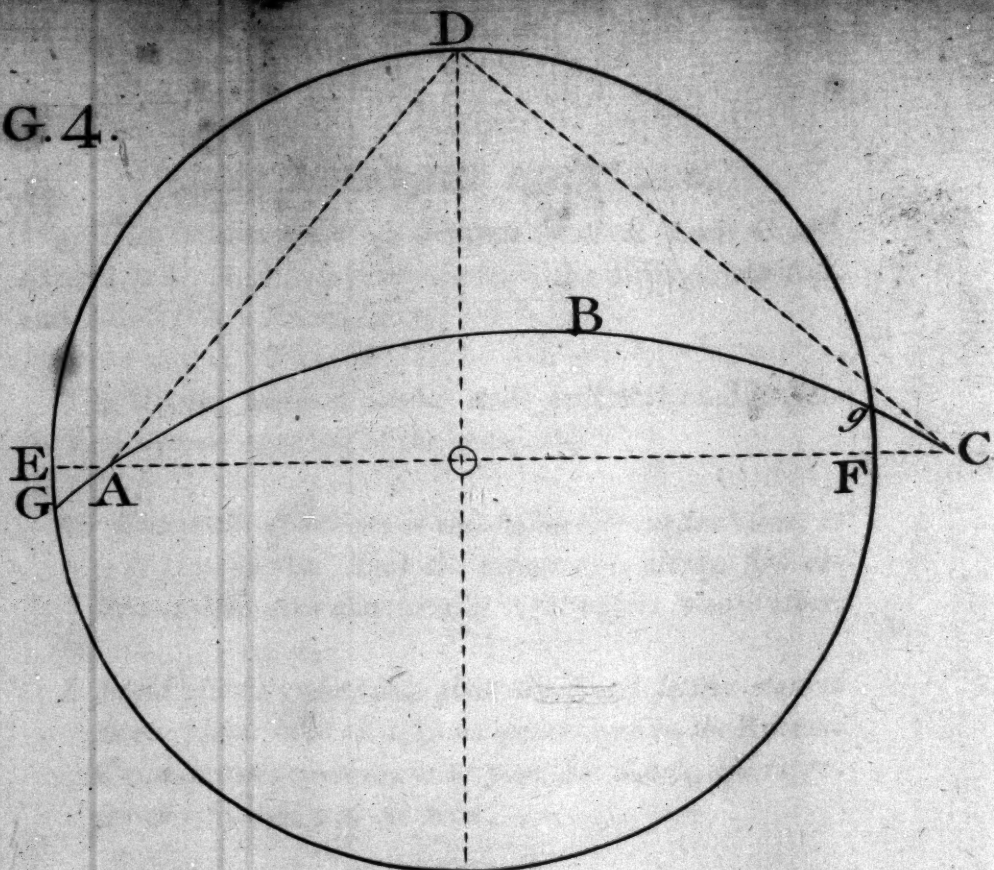
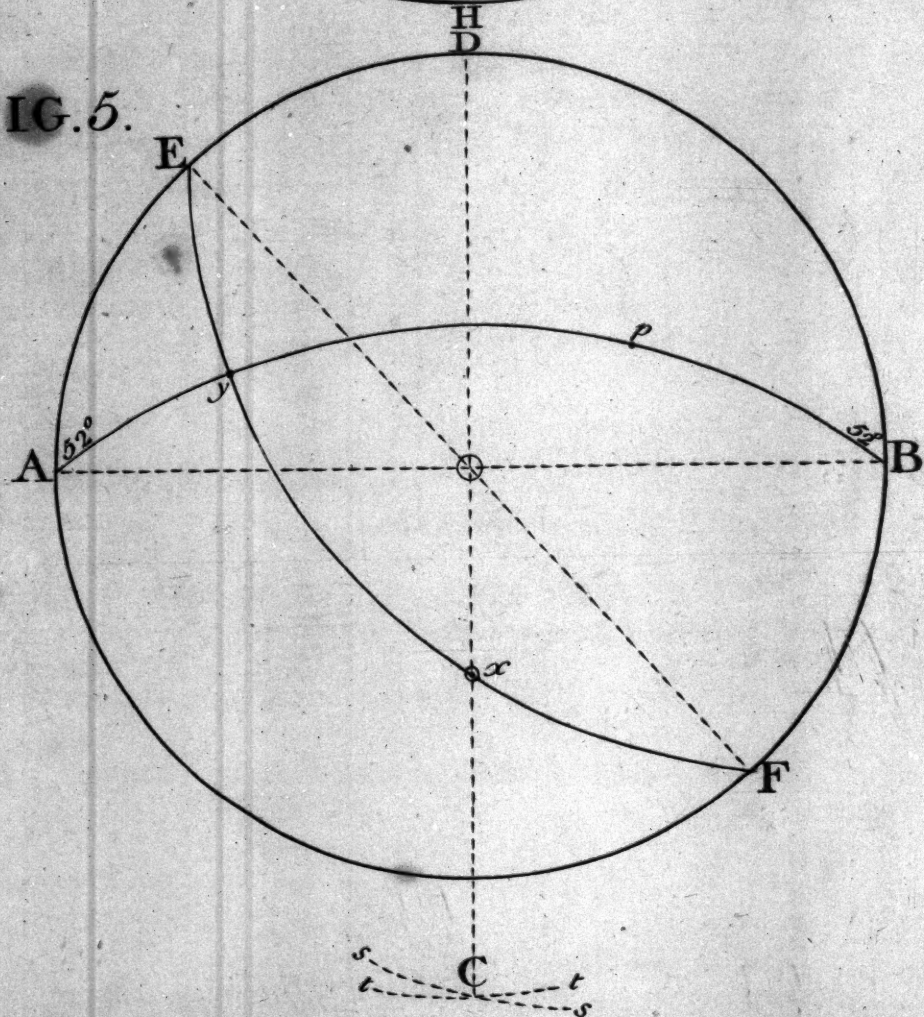


FIG. 5.



PROBLEM VII.—FIG. 4.

Through any two given points (A and B) in the plane of projection, to describe a great circle.

CX. Through either point, suppose A, draw a diameter E A O F; draw H D at right angles, and draw A D; perpendicular to A D draw D C to meet A F produced to C: through A B and C draw a circle, (xcv) then is G A B g the great circle required.

PROBLEM VIII.—FIG. 5.

Through a given point (p) in the plane of projection, to draw a great circle to make a given angle with the primitive, (suppose 52° .)

CXI. With the tan. 52° , and one leg on O, describe the arc tt; with the sec. 52° , and one leg on p, the given point, describe ss, which will cut tt in C, then is C the centre of the oblique circle A p B, which makes the given angle (52°) with the primitive as required. (cix.)

PROBLEM IX.—FIG. 5.

To draw a great circle (E y x F) perpendicular to a given great circle, (A p B.)

CXII. Find the pole x of the given circle, (c) and draw a diameter E O F at pleasure.

Through the points E x and F draw a circle (xcv), and it will be perpendicular to A p B. (xvii.)

If the perpendicular should pass through a given point (y).

Find x (c) and through y and x draw a great circle (cx.)

PROBLEM X.—FIG. 6.

TO PROJECT LESSER CIRCLES.

This problem admits of three cases:

Case 1. *To describe a lesser circle, as F G H I, parallel to the primitive, suppose at 45 degrees distance.*

CXIII. With the semi-tangent 45 degrees, (its complement) describe the circle F G H I, which will be 45 degrees from the primitive, as required.

Case 2. *To describe a lesser circle, as K y l, parallel to the right circle D C E, suppose at 25 degrees distance.*

CXIV. Take the secant of 65 degrees, (its complement) which will extend from C on B produced down to Z, (in Fig. 7) and Z is the centre of K y L.

Then with the tan. 65 on this centre, describe K y L, which is the parallel circle required.—Or,

The chord 25 degrees, laid off from D and E, will give K and L, and the semi-tan. 25 degrees from C will give y.

Then through K y and L describe a circle (xcv.)

Case 3. Fig. 7. *To draw a lesser circle x b m a parallel to an oblique circle A F B, suppose at the distance of 45 degrees.*

CXV. Here if the angle T A E = 55° , take the sum and diff. of 45° and 55° , which is 100° and 10° ; lay these off from the scale of sem. tan. and they will extend from C to b, and to a, bisect the distance ba, which will give C the centre of x b m a, the lesser circle required: thus, $C F + C b = 45^\circ$. (LXXIII). Or thus, a ruler on B and p (the pole of A F B) will give x.

The comp. 45° from x both ways, will give v and y; draw B y and B v, (continued to a) and these will give b and a in the line of measures.

P R O.

FIG. 6.

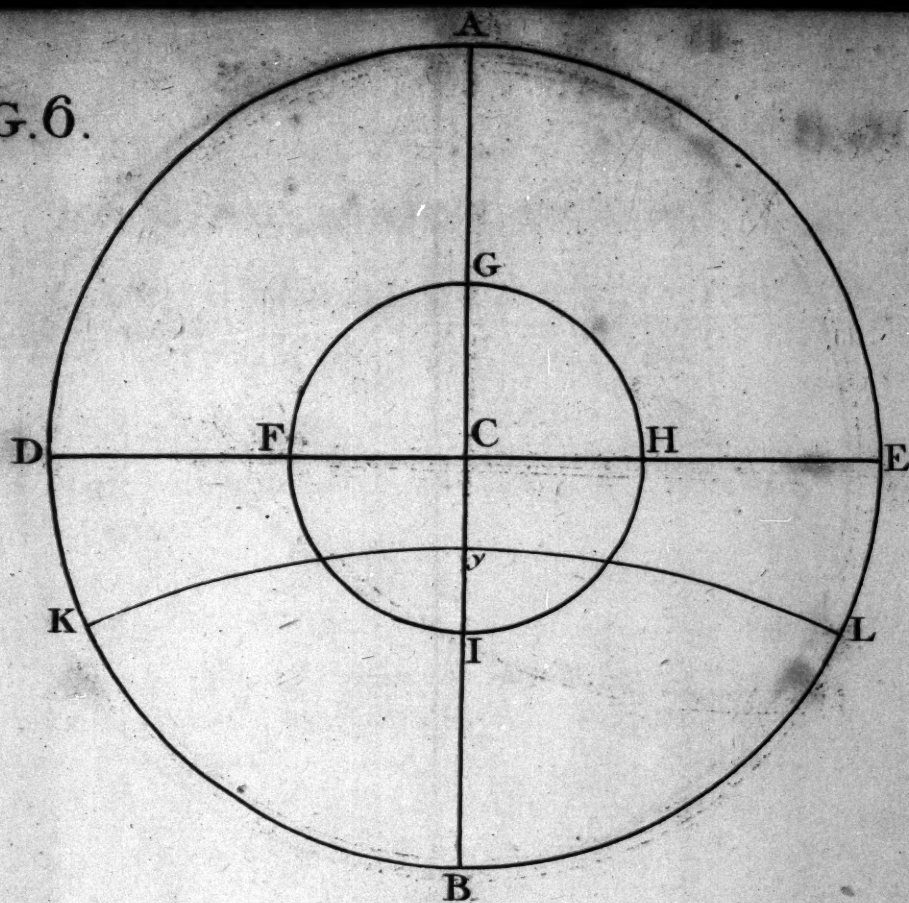


FIG. 7.

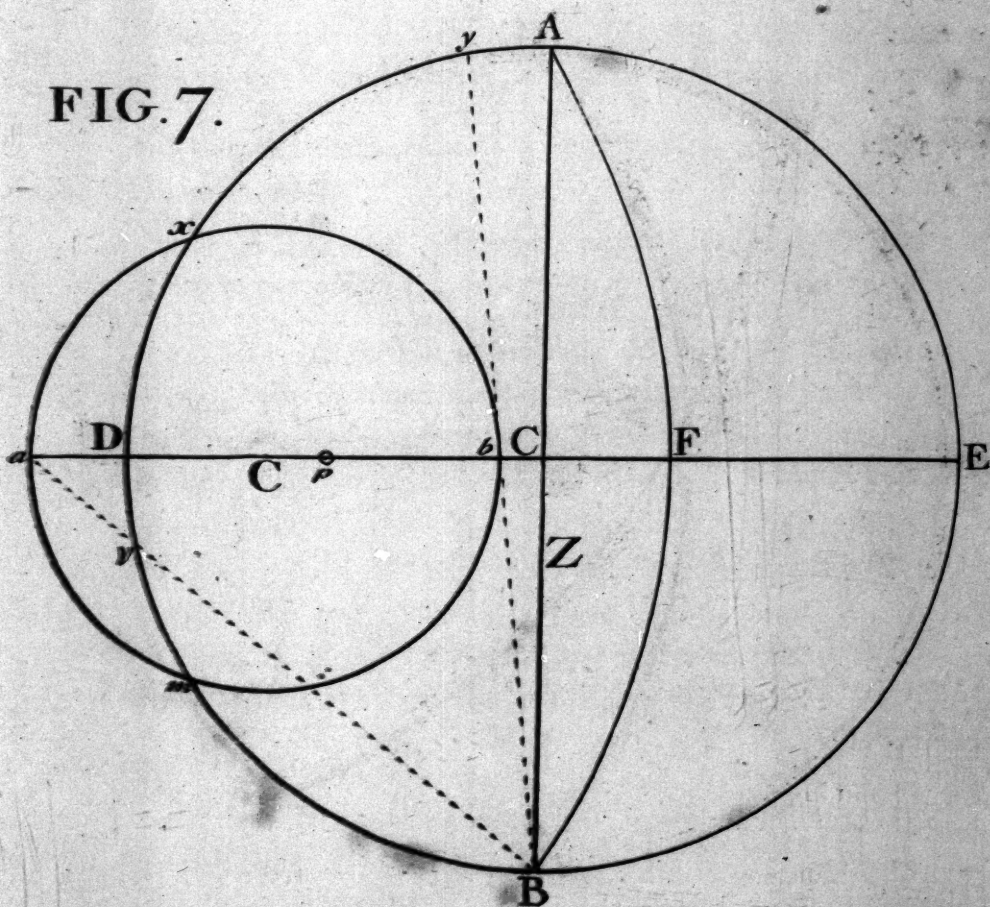


FIG. 8.

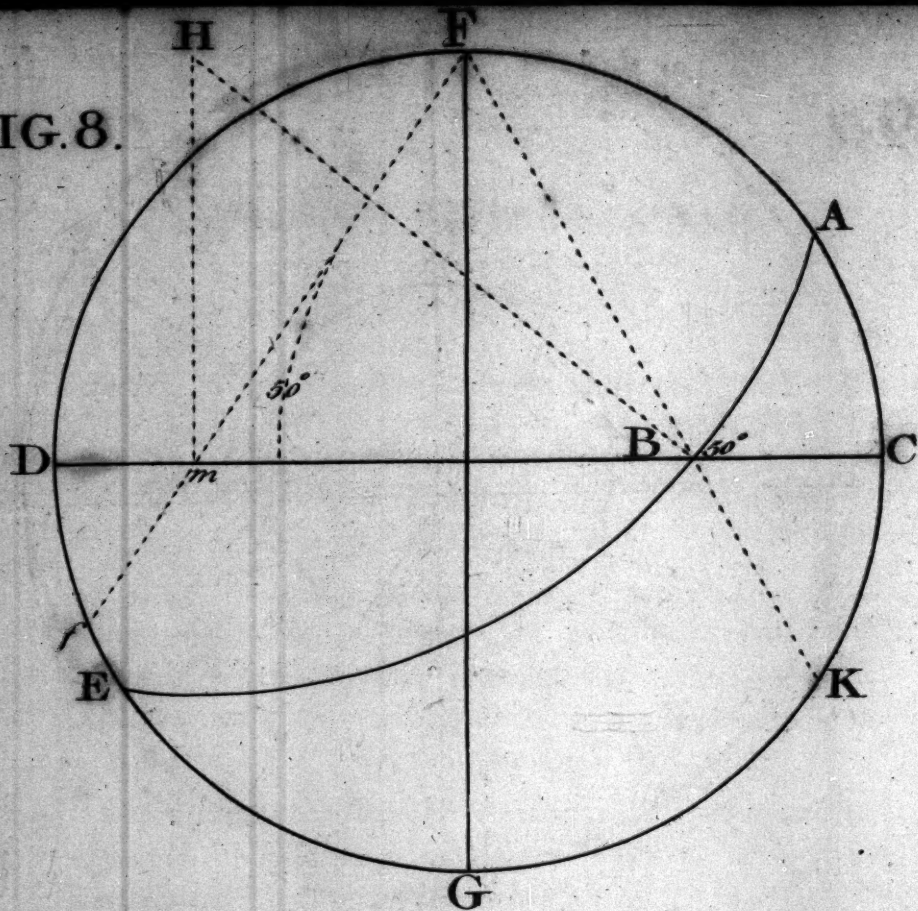
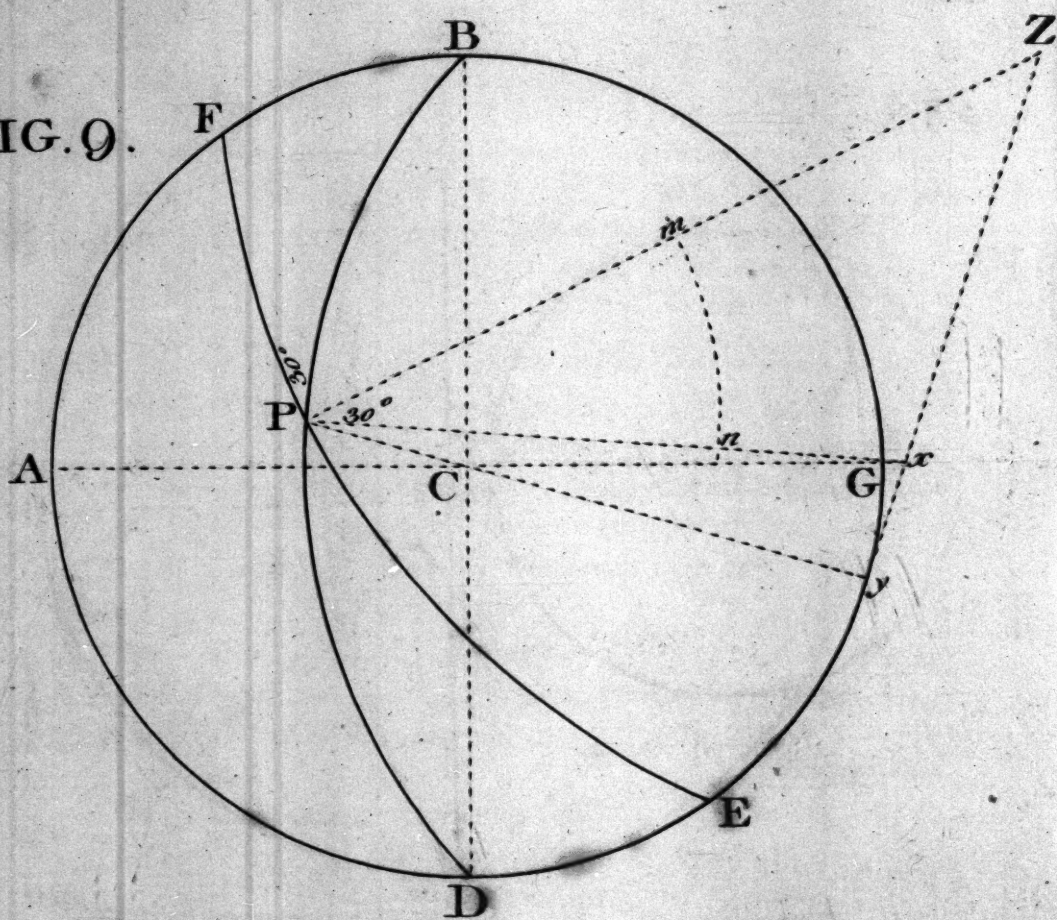


FIG. 9.



PROBLEM XI.

TO PROJECT ANY SPHERIC ANGLE.

This admits of four cases.

Case 1. *When the angle is at the centre, it is laid off at the primitive (cf).*

Case 2. *When the angle is at the primitive, it is laid off, as in Art. cix.*

Case 3. (FIG. 8). *When the angle is formed by a right and oblique circle, suppose to make $ABC = 50$ degrees.*

CXVI. From the point F, through the given or assumed point B, draw a right line F B K.

Lay off $\frac{1}{2}$ CK from G to f, draw f F, and mark where it cuts D C in m.

From m erect the perp. m H, and make an angle m B H, equal to the comp. of the proposed angle ($=40$).

Then the point H will be the centre of the oblique circle A B C.

Case 4. (FIG. 9). *When the angle is formed by the intersection of two oblique circles, suppose the angle should be 30° .*

CXVII. Describe an oblique circle B P D with any radius, suppose P x; in this circle assume a point as P from whence draw two lines P x and P C continued so as to meet a perp. let fall from x to P C y, and let y x be produced at pleasure. From P lay off the plane angle m P n = the proposed angle: let P m be continued to meet y x in Z.

Then is Z the centre of the oblique circle E P F, which makes angles of 30° with D P B.

PROBLEM XII.

TO DESCRIBE A GREAT CIRCLE TO MAKE A PROPOSED ANGLE WITH A GIVEN GREAT CIRCLE, AND WITH THE PRIMITIVE.

This admits of two cases.

Case 1. (FIG. 10). *When the given circle is a right one, as C D, suppose to make the angle at the intersection 42° , and that at the primitive 64° .*

CXVIII. About O, the pole of the primitive with semi-tangent 64° describe m n (CXIII).

About A, the pole of the right circle C D, describe x y at the distance 42° , (CXIV).

The intersection z of these two lesser circles is the pole of F G E, the required oblique circle, which is described with the secant 64° , (CIX).

Case 2. (FIG. 11). *When the given circle is an oblique one, as A n B, to make angles as above.*

CXIX. About the pole of the primitive, with semi-tangents 64° , describe v x (CXIII).

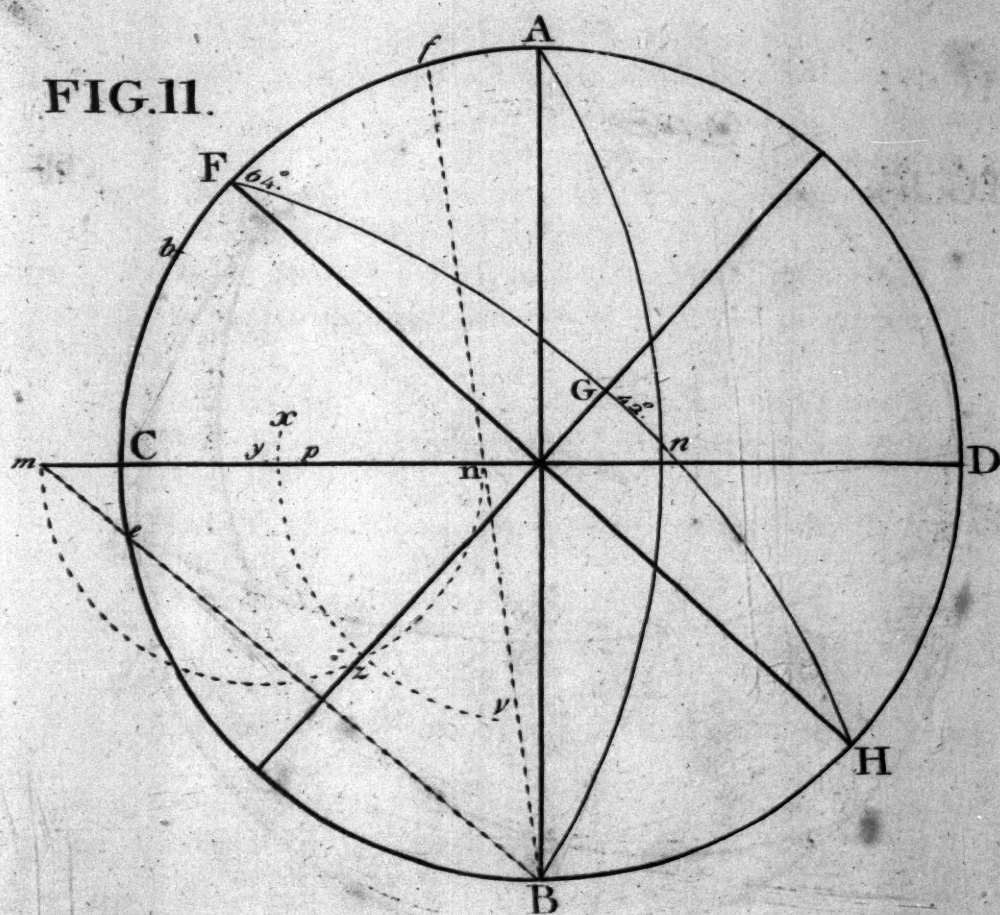
About p, the pole of the given circle, (c) describe a parallel circle m z n, at the distance 42° from p (CXV); then is Z the pole of the required circle F G n H, which is described with the Sec. 64° (CIX).

To make this operation still plainer, the manner of drawing mzn (CXV), is here repeated.

A ruler on B and p gives b, and 42° laid off both ways from b, will give e and f.

A ruler on b and e gives m, and on B and f gives n, bisect m n, which gives y, the centre of m z n.

P R O-

[illegible]

P R O B L E M XIII.

TO CUT A GREAT CIRCLE IN A GIVEN ANGLE, AND HAVE A GIVEN ARC INTERCEPTED BETWEEN THE POINT OF INTERSECTION AND THE PRIMITIVE.

This admits of two cases.

Case 1. (FIG. 12). *When the given circle is a right one, as E A, to cut it so as to make the angle C B A = 50° , and the arc C B = 48° .*

CXX. Lay off D O H = 40° , the complement of the proposed angle.

Make O H the tangent 48° , the proposed arc.

With the Secant of the same arc from O, describe s S.

Through H, draw H S, parallel to E A.

From the point of intersection S, with the tangent H O, describe the arc T t; which will cut E A in B.

Draw a right line through S, and the centre O, cutting the primitive in C and L.

Through L, B and C, draw a circle (xcv), then is the angle C B A = 50° , and the arc C B = 48° , as required.

Case 2. (FIG. 13). *When the given circle is an oblique one, as E D A, suppose the angle e x A, should equal 44° , and the arc e x = 57° .*

CXXI. Lay off the angle B D H = 46° , (the comp. of the proposed angle) make D H = the tan. of the proposed arc (57°).

From O, with Sec. 57° , describe S s, and from Z (in D B continued) the centre of E D A, with radius Z H cut S s in s.

From s with tan. D H, draw t t, cutting the circle E D A in x.

Through s, and the centre O, draw s e O L.

Through L, x and E, draw a circle (xcv), then will the angle e x A = 44° , and the arc e x = 57° .

Thus m n = 44° , and e r = 57° (cvii and ciii).

THE TWO FOLLOWING SECTIONS shew the manner of projecting spheric triangles, which may be laid off either with an angle at the projecting point, or with a side upon the primitive; but the latter is preferable, as affording most room for the figure.

Spheric Triangles may be represented upon a globe by the help of a quadrant of altitude;—thus, to represent Fig. 14, A B and A C being given = 55° and $63^{\circ} 56'$.

Let the inner edge of the wooden horizon represent the primitive circle, and the brazen meridian the right circle E B C.

From the notch at C mark 55° to A on the horizon.

Take $63^{\circ} 56'$ on the quadrant of altitude, which will extend from A to B on the meridian; then is B C found = 40° .

The angles are measured by continuing the sides to quadrants, and finding the nearest distances between the extreme points.

SECTION

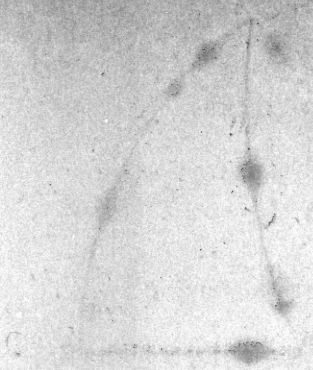


FIG. 1

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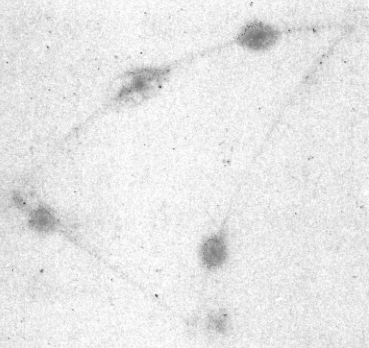


FIG. 2

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FIG.14.

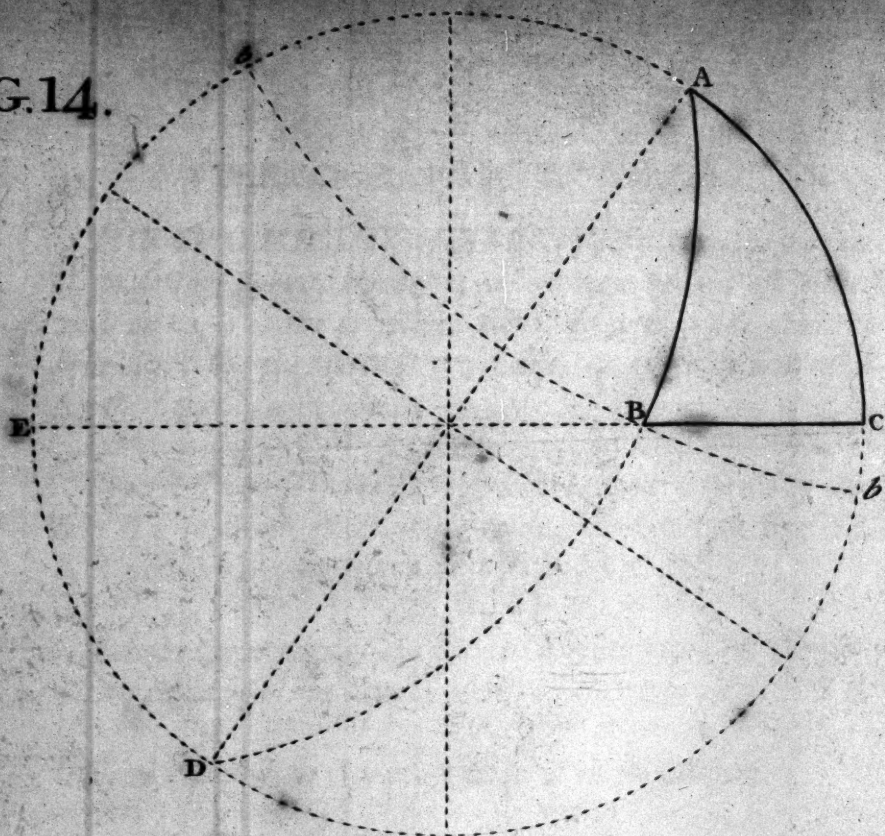
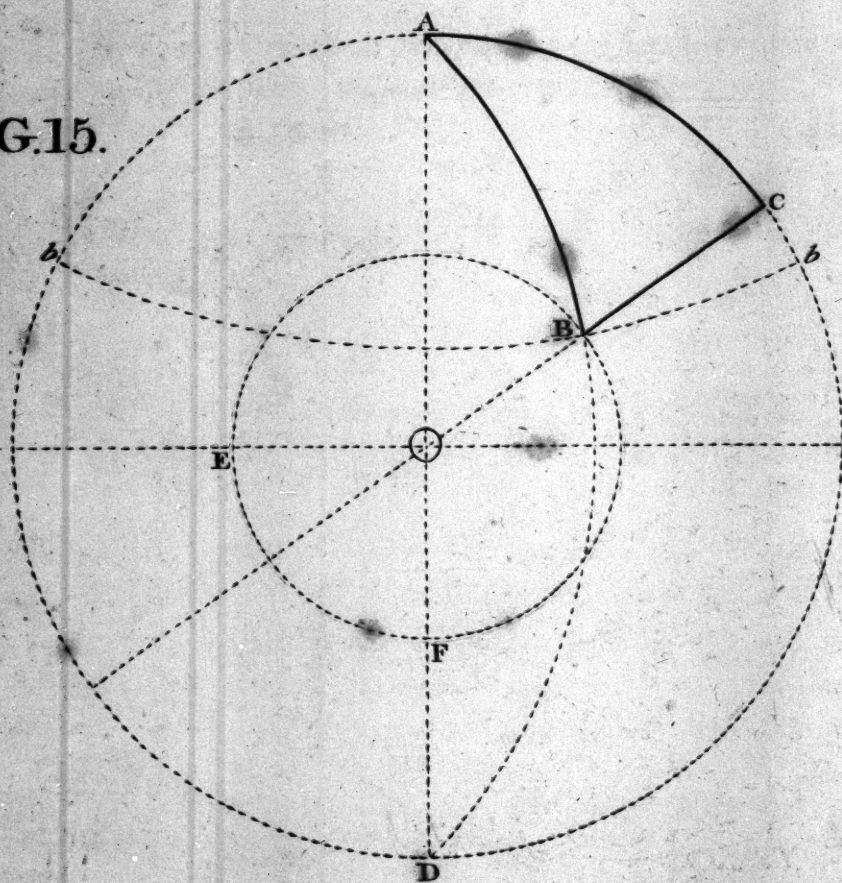


FIG.15.



SECTION XIII.

TO PROJECT RIGHT ANGLED SPHERIC TRIANGLES.

EVERY right angled spheric triangle being composed of six parts, it is necessary that three be given to project the figure, and measure the rest; but as the right angle is always known, it will be sufficient that two other parts be given; it admits of six cases which may be considered as a continuation of the problems.

PROBLEM XIV. CASE 1.

Given $\left\{ \begin{array}{l} \text{the Hyp. } AB = 63^\circ 56' \\ \text{A Leg } AC = 55^\circ \end{array} \right\}$ (Fig. 14) To construct the triangle.

CXXI. Lay off the given leg AC on the primitive $= 55^\circ$, draw the diameter AD , and about A , as a pole at a distance equal to the hyp. $= 63^\circ 56'$, describe the parallel circle bb (cxiv), which will cut the right circle EC in B .

Through A , B and D , describe a circle (XCV), and ABC is the triangle required.

To put the given leg on the right circle (supp. 40°) and the required leg on the primitive, (Fig. 15).

CXXIII. About any point in the primitive, supp. A , describe the lesser circle bb , at the distance of AB , the hyp. (cxiv).

About the centre O , describe the lesser circle, EBF (cxiii), at the distance 50° , (the comp. of BC).

Through the point of intersection B , draw the diameter, and through A , B and D , draw a circle (xcv); then is ABC , the triangle required.

PROBLEM XV.—CASE 2.

Given $\left\{ \begin{array}{l} \text{the Hyp. } AB = 63^\circ 56' \\ \text{the angle } A = 45^\circ 41' \end{array} \right\}$ (Fig. 16) To construct the triangle.

CXXIV. Describe an oblique circle ABD , to make an angle $45^\circ 41'$, with the primitive (cix).

About A , describe the lesser circle, bb ; at the distance $63^\circ 56'$ (cxiv).

Through B the point of intersection, draw the diameter BC ; then is ABC the triangle required.

Suppose it were required to lay off the given angle at B , (Fig. 17).

CXXV. This is performed by Prob. 13, Case 1; to make the angle $ABC = 45^\circ 41'$, and to have the intercepted arc, $AB = 63^\circ 56'$.

This triangle, as well as the others in this and the following Section, is measured by Problems 4th and 5th.

Thus the pole p of AB y D being found (c). To measure $\angle B$.

A ruler on B and p , will give q in the primitive; and on B and F will give F .

Then is $qF = 45^\circ 41' =$ the angle B .

$AC = 40^\circ$, (Lxiv) and $BC = 55^\circ$ (Lxxiv).

A ruler on p and B will give r in the primitive (ciii), then is $rA = 63^\circ 56' =$ the arc Ar .—Also

A ruler on A and p will give n ; then is $nD = 65^\circ 46' =$ the angle A (cv); or, semi-tan. $Op = \angle A$, (cvi) or semi-tan. $yZ = \angle A$, (cvii).

PRO-

FIG.16.

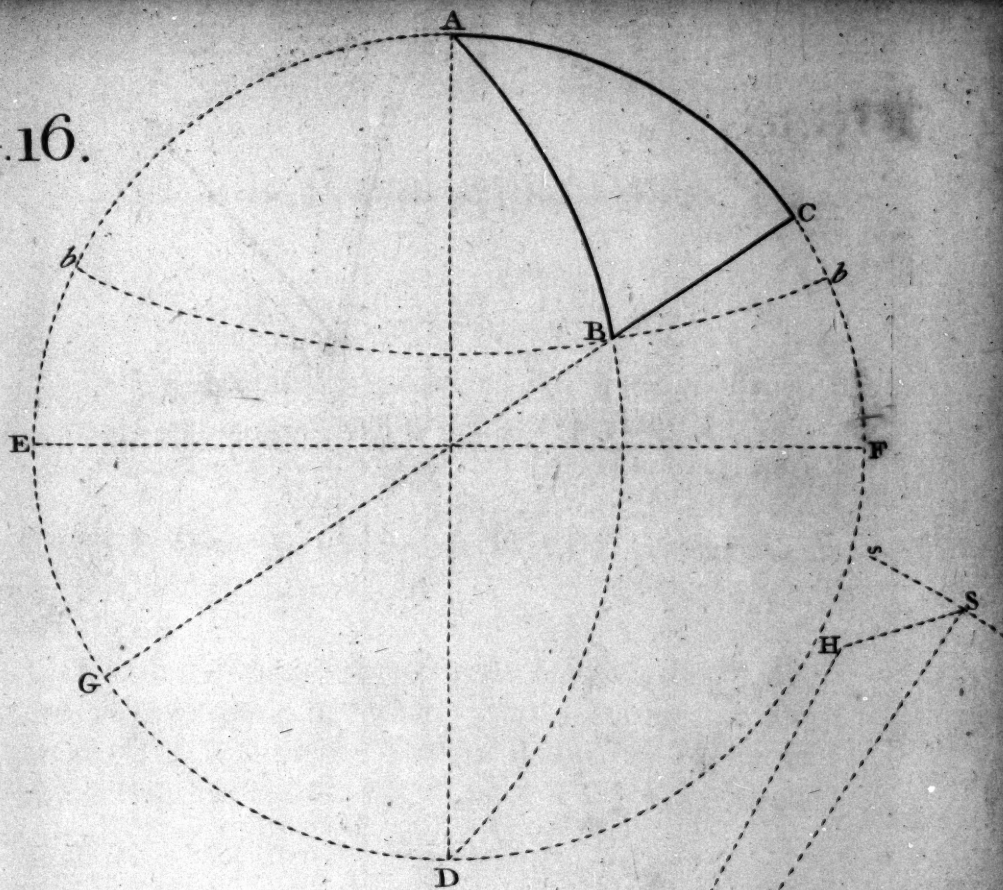


FIG.17.

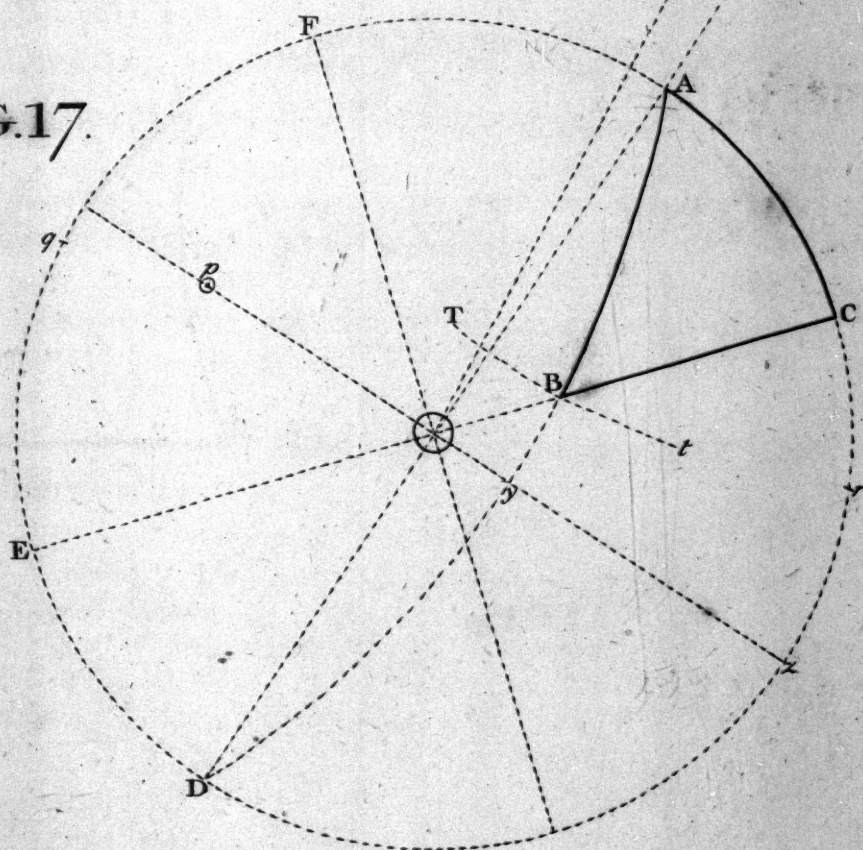


FIG.18.

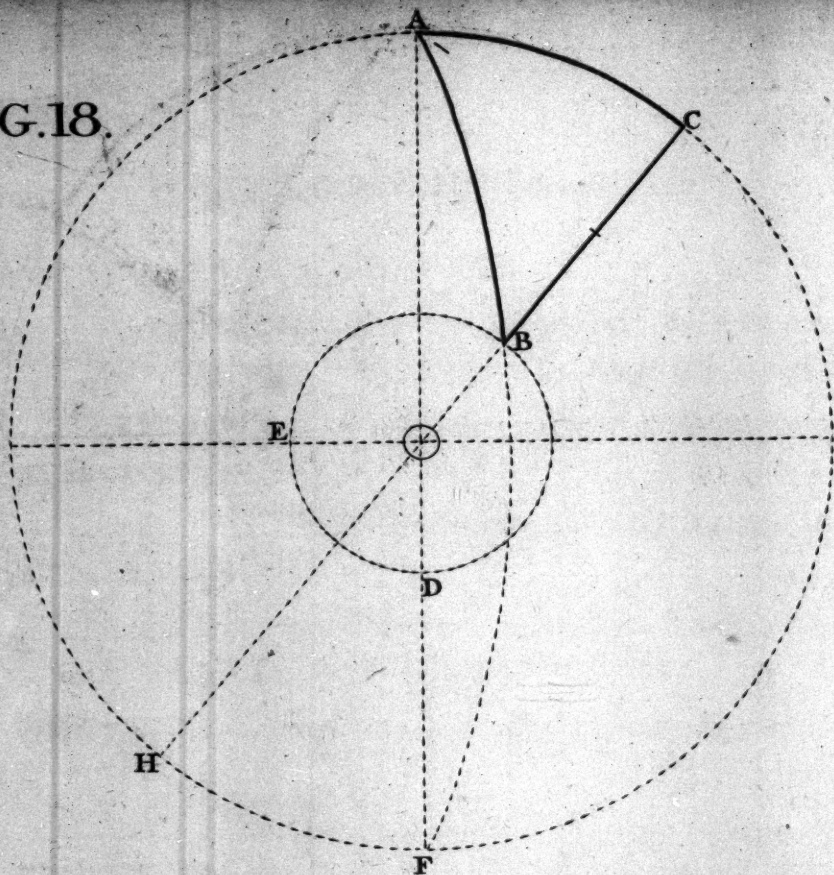
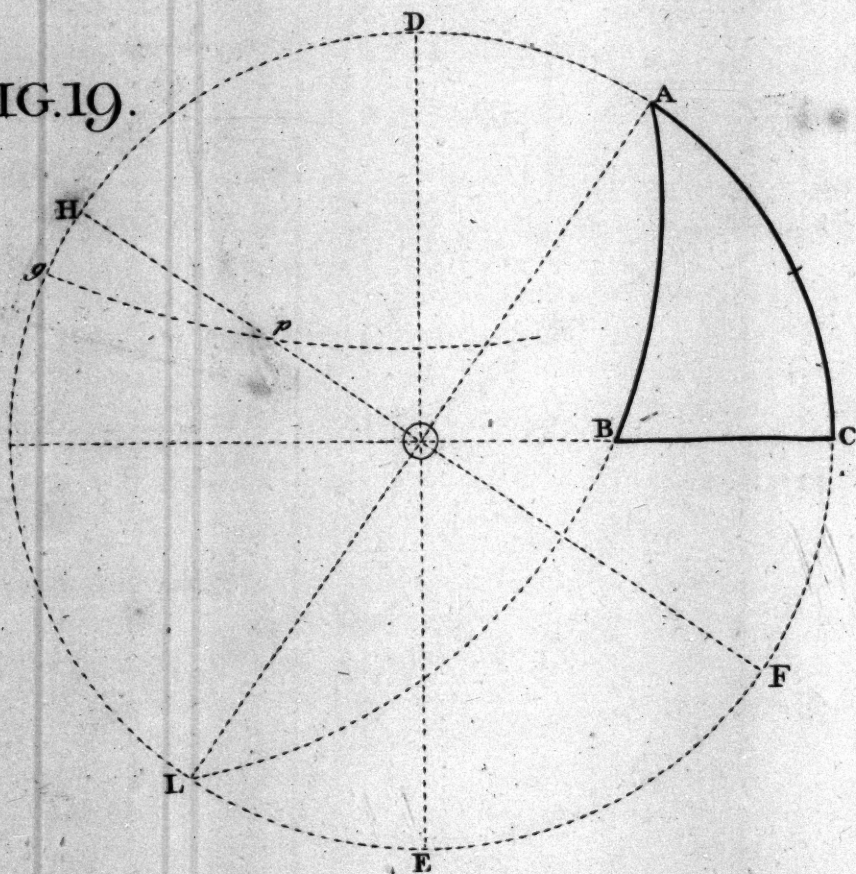


FIG.19.



PROBLEM XVI.—CASE 3.

Given $\left\{ \begin{array}{l} \text{A leg } BC = 55^\circ \\ \text{and its opp. angle} = 65^\circ 46' \end{array} \right\} \begin{array}{l} \text{to construct} \\ \text{the Figure,} \\ \text{\&c. (Fig. 18.)} \end{array}$

CXXVI. Draw the oblique circle A B F, to make the angle at A = $65^\circ 46'$. (CIX.)

Then with the comp. of the oppos. leg B C, (35°) describe the parallel circle E D B, (CXIII) cutting the oblique circle A B F in B; through B and the centre O draw the right circle H O B C, and A B C is the triangle required.

CXXVII. *If it were required to put the given leg upon the primitive.* (Fig. 19.)

From C to A lay off the chord 55° , and draw the diameters A O L and H O F at right angles.

About D, at the distance of the proposed angle, $65^\circ 46'$, describe the parallel circle g p. (CXIV.)

The point p, where this parallel circle cuts the diameter H O F in p, is the pole of the oblique circle A B L.

Hence the pole being found, the oblique circle may be thus described:

Measure p O, (LXXIII) and the sec. of this is the radius of the oblique circle A B L. (CIX.)

P R O-

PROBLEM XVII.—CASE 4.

Given $\left\{ \begin{array}{l} \text{A leg } AC = 55^\circ \text{ and} \\ \text{its adjacent angle } A = 45^\circ 41' \end{array} \right\}$ to construct the figure. (Fig. 20.)

CXXVIII. From C to A lay off the chord 55° ; draw the diameter A G with another at right angles.

With the secant $45^\circ 41'$ draw the oblique circle A B G. (cix.)

Then is A B C the triangle required.

CXXIX. Suppose the leg B C were given $= 40^\circ$, and its adjacent angle B $= 65^\circ 46'$.—Fig. 21.

Make the leg B C $= 40^\circ$. (LXXIV.)

Then, through the point B describe the oblique circle A B D, to make the angle A B C $= 65^\circ 46'$ (CXVI).

Thus, from E through the point B draw E B F; lay off 2° C F; from G to h draw h E; from the point m where h E cuts the diameter, erect a perpendicular m x.

Lay off the angle m B x $= 24^\circ 14'$ the comp. of the required angle, which will give x, the centre required.

P R O.

FIG. 20.

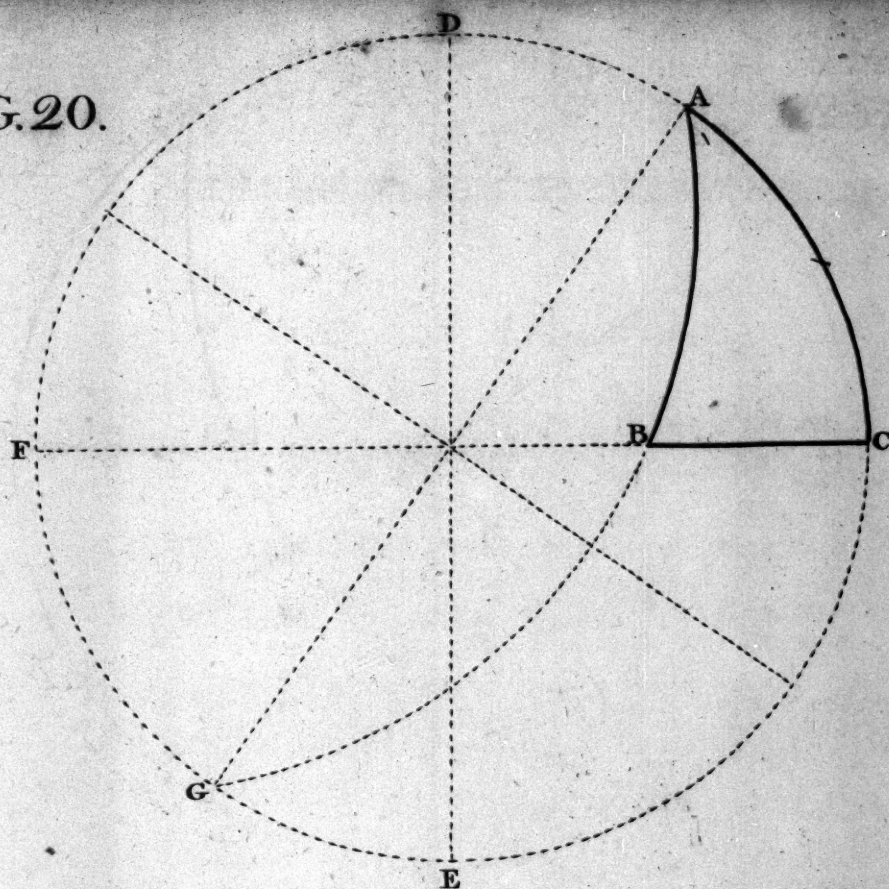


FIG. 21.

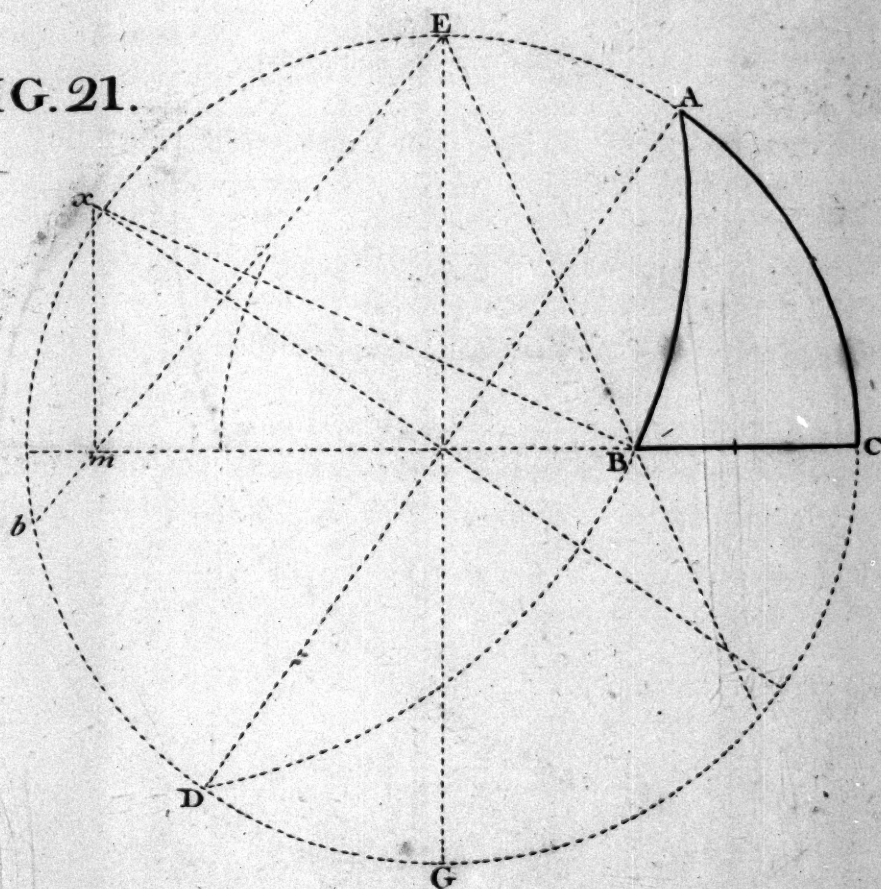


FIG. 22.

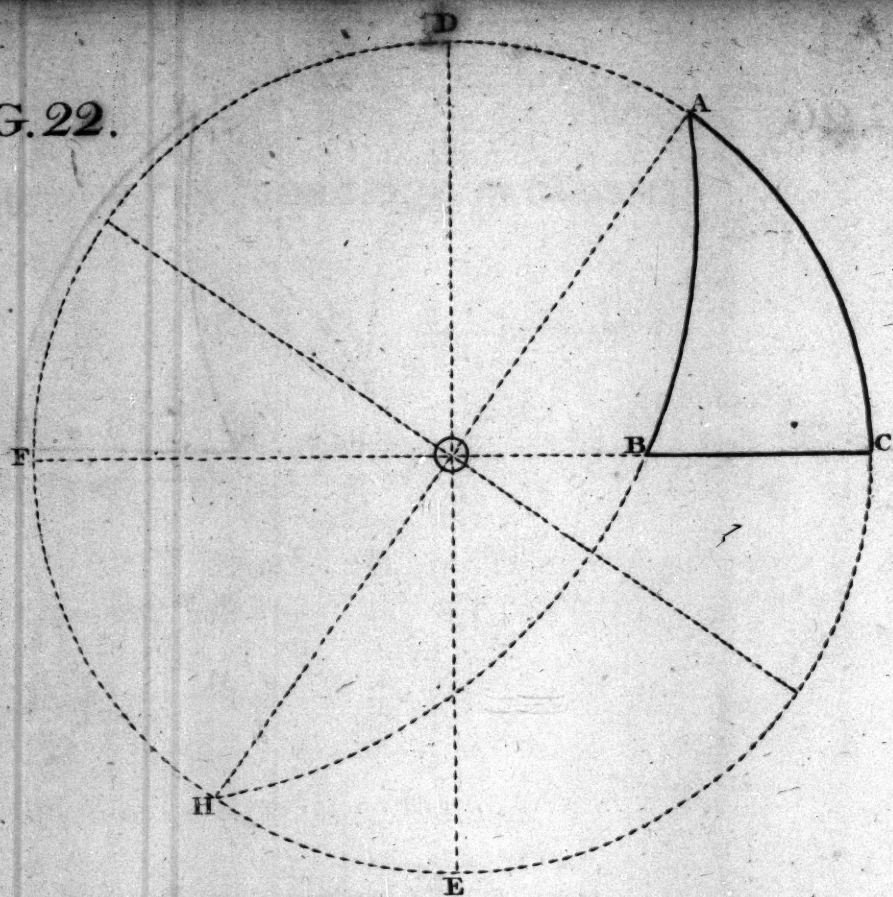
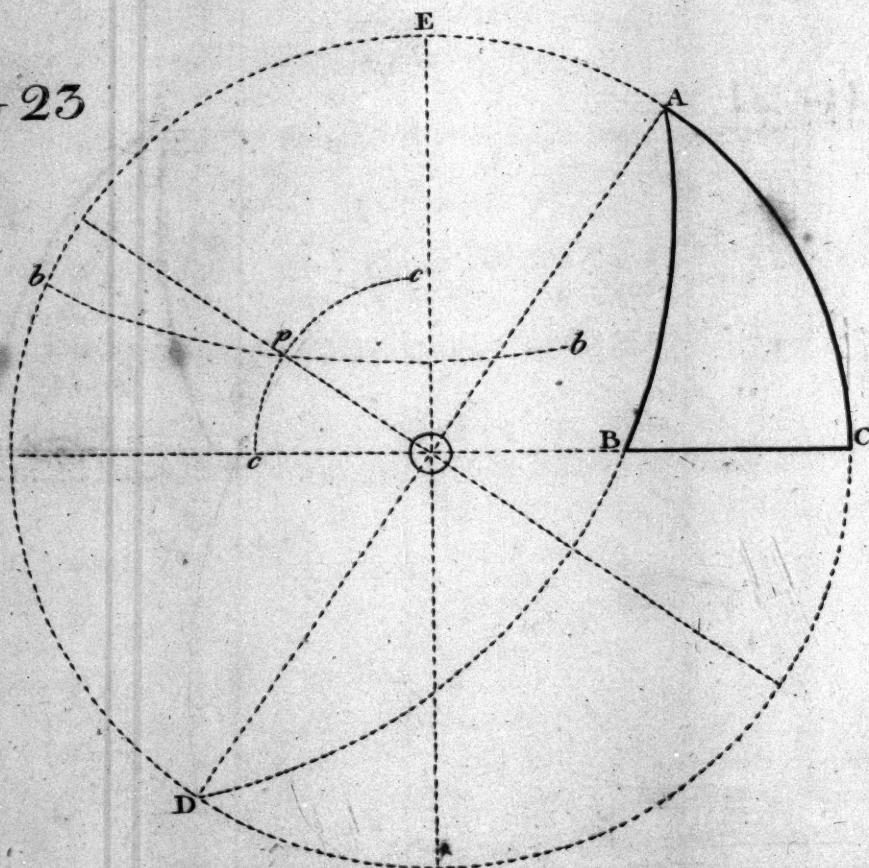


FIG 23



PROBLEM XVIII.—CASE 5.

Given $\left\{ \begin{array}{l} \text{the leg } A C = 55^\circ \\ \text{the other leg } B C = 40^\circ \end{array} \right\}$ to construct the figure. (Fig. 22.)

CXXX. Lay off $C A =$ the chord 55 degrees.

And $B C =$ the semi-tan. 40 degrees. (LXXIV.)

Draw the diameter $A O H$, and through the three points $H B A$ describe the oblique circle, (xcv) and $A B C$ is the triangle required.

PROBLEM XIX.—CASE 6.

Given $\left\{ \begin{array}{l} \text{the angle } A = 45^\circ 41' \\ \text{the angle } B = 65^\circ 46' \end{array} \right\}$ to construct the figure. (Fig. 23.)

With the semi-tan. $45^\circ 41'$ and one foot in O , describe the arc $c c$. (CXIII.)

About E , at the distance $65^\circ 46'$, describe $b b$. (CXVI.)

Where these two parallel circles cross in p is the pole of the oblique circle $A B D$.

Through O and p draw a diameter, and with the secant $45^\circ 41'$, describe $D B A$, and $A B C$ is the triangle required.

PRO-

SECTION XIV.

TO PROJECT OBLIQUE SPHERIC TRIANGLES.

This, like the projection of right-angled spheric triangles, admits of six cases—viz. when

- | | |
|------------------------------------|--------------|
| 1. Two sides and an opposite angle | } are given. |
| 2. Two sides and an included angle | |
| 3. Two angles and an opposite side | |
| 4. Two angles and an included side | |
| 5. Three sides | |
| 6. Three angles | |

PROBLEM XX.—CASE I.

CXXXII. Given two sides and an opposite angle.—
viz.

$AC = 58^\circ$	} To construct the figure, and find the rest,
$BC = 110$	
$\angle BAC = 121^\circ 55'$	

From C to A, lay off 58° , and draw the diameter, AOD.

Draw the oblique circle DBA, making the angle $BAE = 58^\circ 5'$, the supplement of $121^\circ 55'$.

Then to lay off the side $BC = 110$.

About E describe the parallel circle p g, at the distance of 70° , (the supplement 110°).

Through

FIG. 24.

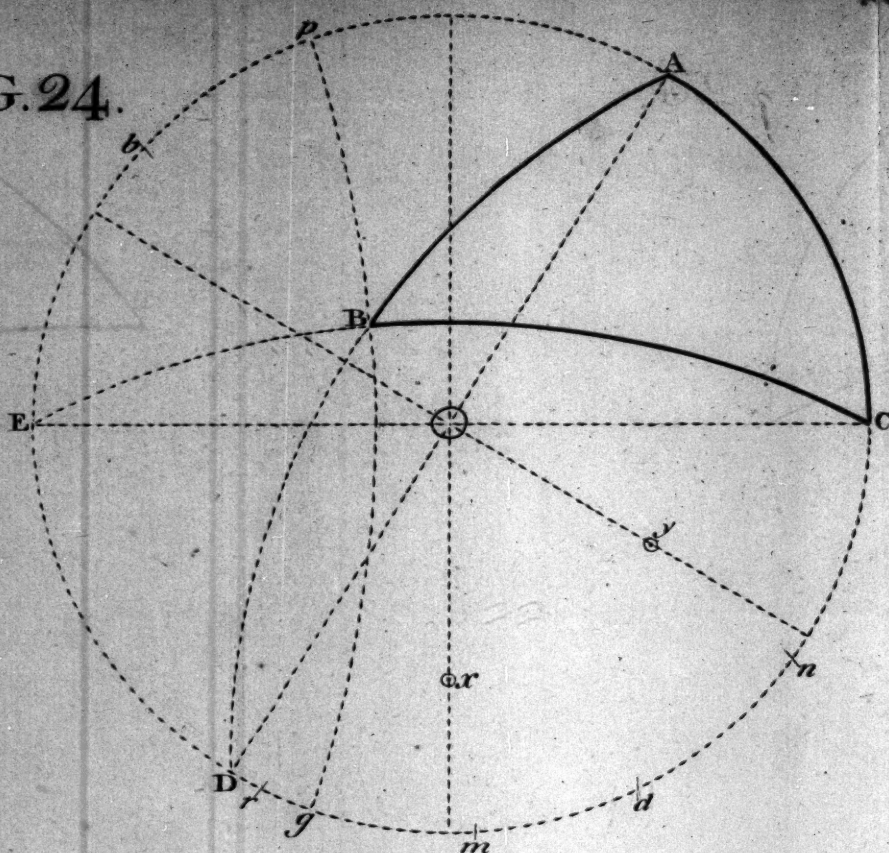


FIG. 25.

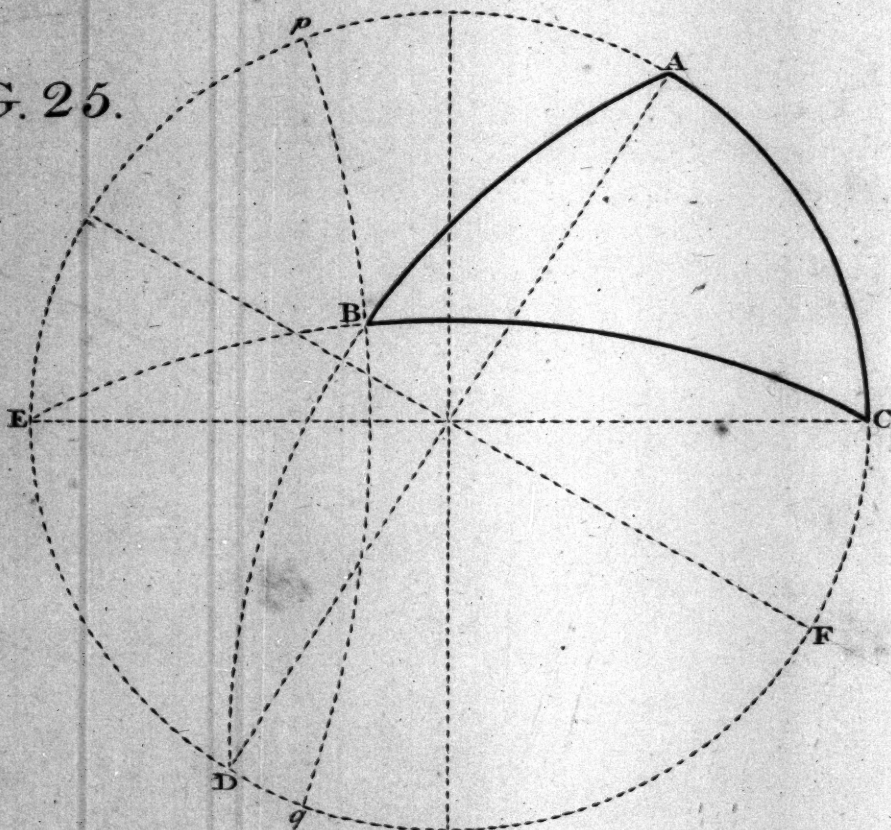


FIG. 26.

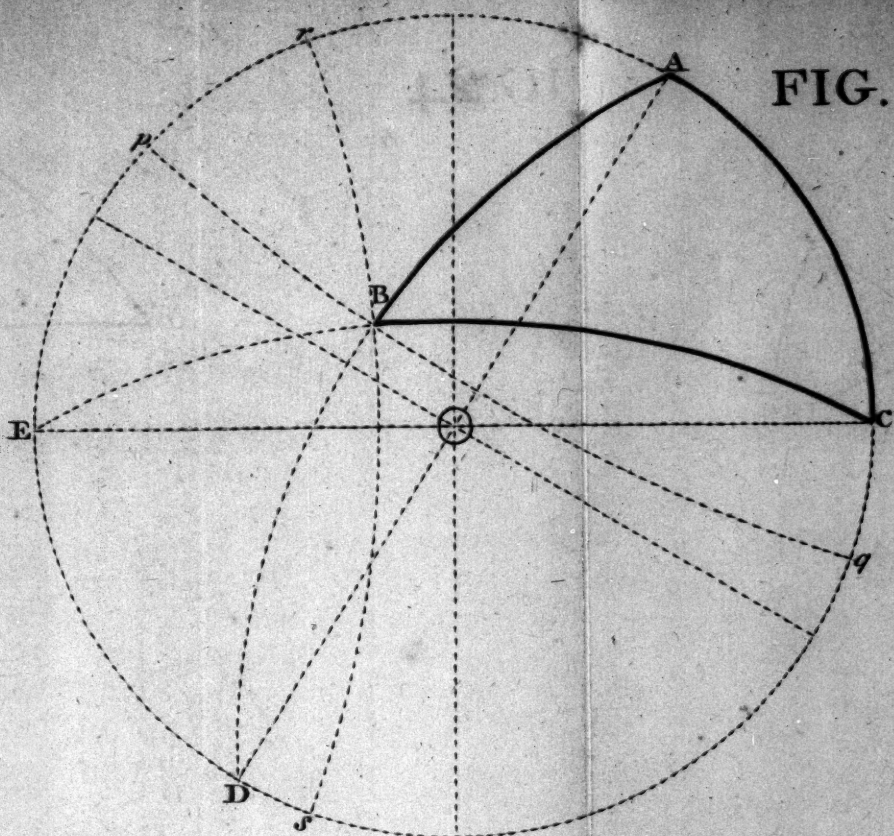
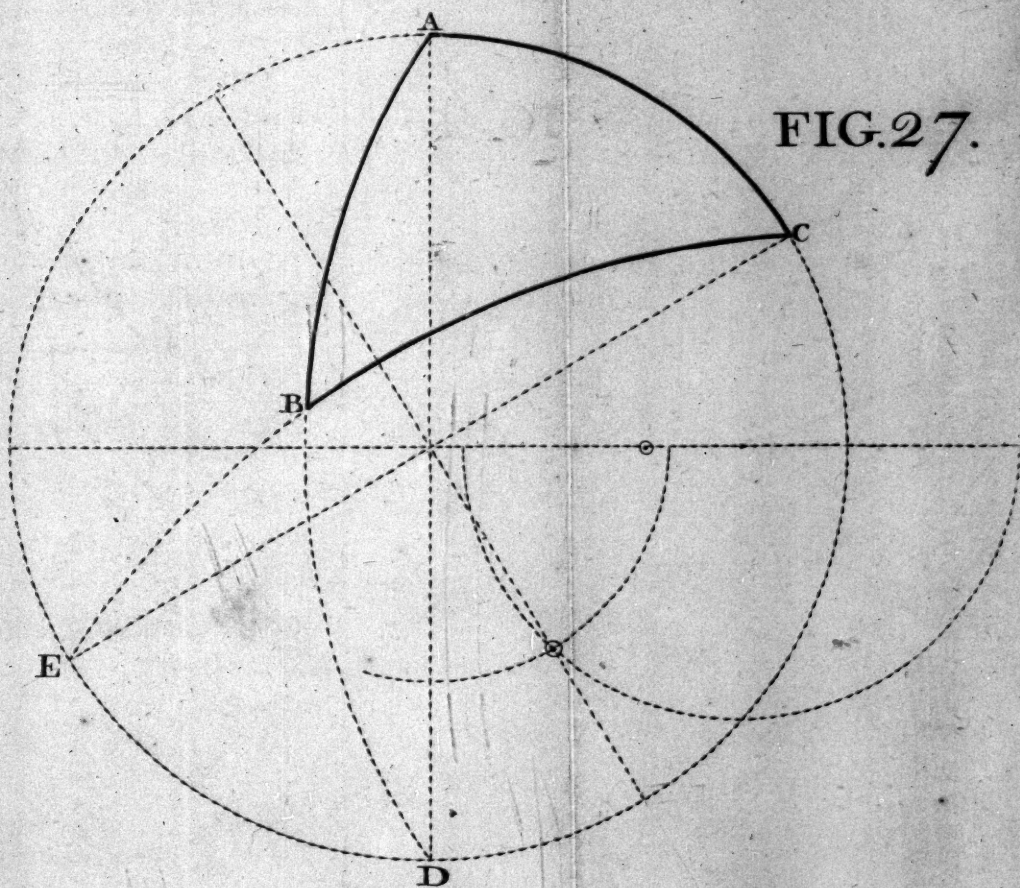


FIG. 27.



Through the point B (where the lesser circle pg crosses the oblique circle A B D), and the points E and C describe a circle (xcv), and A B C is the triangle required.—See Fig. 24.

To measure the sides and angles. Find the poles x and y of the two oblique circles E B C and A B D (c).

The side A C, is measured on the scale of chords $\approx 58^\circ$.

The side B A is found by laying a ruler on y and B, which will cut the primitive in b; then is b A the measure of the side B A $\approx 79^\circ 17'$.

The side B C is found by laying a ruler on x and B, which will give p in the primitive; then is p c the measure of the side B C $\approx 110^\circ$.

To measure the angle A.—A ruler from A on the pole y gives d, and on the pole O, gives D; then is D d the measure of A; that is, its supplement (cvi.) $\approx 121^\circ 55'$.

In the same manner m n is found the measure of the angle B $\approx 50^\circ$, and E r is the measure of $\angle C \approx 62^\circ 34'$.

PROBLEM XXI.—Case 2.

Given $\left\{ \begin{array}{l} \text{the side A C} = 58^\circ \\ \text{the side B C} = 110^\circ \\ \text{the included } \angle C = 62^\circ 34' \end{array} \right\}$ to construct the figure, and find the rest. (Fig. 25.)

E

CXXXIII. Draw

STEREOGRAPHIC PROBLEMS.

CXXXIII. Draw the oblique circle EBC , to make the angle at $C = 62^\circ 34'$. (cix.)

Lay off the cho. 58° from C to A ; round E , at the distance of 70° , the supplement of 110 , draw the parallel circle pq , (cxiv), which will cut the oblique circle in B .

Draw the diameter AD , and through D , B , and A , describe a circle, (xcv); and ABC is the triangle required.

PROBLEM XXII.—CASE 3.

Given $\left\{ \begin{array}{l} \text{the angle } A = 121^\circ 55' \\ \text{the angle } C = 62^\circ 34' \\ \text{the opposite side } BC = 110^\circ \end{array} \right\} \begin{array}{l} \text{to con-} \\ \text{struct the} \\ \text{figure,} \\ \text{\&c. (Fig. 25.)} \end{array}$

CXXXIV. Draw the oblique circle EBC , and the parallel circle pq , as in the last case.

Then through the point B describe the oblique circle DBA , to make the angle $BAE = 58^\circ 5'$, the sup. of the given angle BAC ; (cix) and ABC will be the angle required.

PROBLEM XXIII.—CASE 4.

Given $\left\{ \begin{array}{l} \text{the angle } A = 121^\circ 55' \\ \text{the angle } C = 62^\circ 34' \\ \text{the included side } AC = 58^\circ \end{array} \right\} \begin{array}{l} \text{to con-} \\ \text{struct the} \\ \text{figure.} \\ \text{(Fig. 25.)} \end{array}$

CXXXV. Draw

STEREOGRAPHIC PROBLEMS. 51

CXXXV. Draw the oblique circle EBC, and the parallel circle pq, as in Art. CXXXIII.

From the scale of chords lay off 58° from C to A; draw the diameter AD, and through the points A, B, and D, describe a circle, (xcv) then is ABC the triangle required.

PROBLEM XXIV.—CASE 5.

Given $\left\{ \begin{array}{l} \text{the side AB} = 79^\circ 17' \\ \text{the side BC} = 110^\circ \\ \text{the side AC} = 58^\circ \end{array} \right\}$ to construct the triangle. (Fig. 26.)

CXXXVI. From the scale of chords lay off 58 degrees from C to A; about A, at the distance $78^\circ 39'$, describe a parallel circle pq, (cxiv.)

In like manner describe the parallel circle rs about E, at the distance 70° , (the supp. of 110° .)

Where these two parallel circles cross in B is the angular point.

Draw the diameter AD, and through the points D, B, and A, describe a circle, (xcv.)

Then is ABC the triangle required.

PROBLEM XXV.—CASE 6.

Given $\left\{ \begin{array}{l} \text{the angle A} = 121^\circ 55' \\ \text{the angle B} = 50^\circ \\ \text{the angle C} = 62^\circ 34' \end{array} \right\}$ to construct the triangle. (Fig. 27.)

E 2

CXXXVII. Draw

CXXXVII. Draw the oblique circle ABD , (cix), making the angle $BAE = 58^{\circ} 5'$, (the sup. of $121^{\circ} 55'$).

Then by Art. cxix.

Describe the great circle $EB C$, to make the angle $ABC = 50^{\circ}$, and the angle at the primitive $BCA = 62^{\circ} 34'$.

Then is ABC the triangle required.

SECTION XV.

FIG. 28.

AN EASY METHOD OF SOLVING CERTAIN CASES OF SPHERIC TRIANGLES BY THE LINE OF CHORDS ONLY, WITHOUT PROJECTING THE TRIANGLE.

THIS method applies but to four cases, namely,

- | | |
|---|--------------|
| 1. When three sides | } are given. |
| 2. When three angles | |
| 3. When two sides and an included angle | |
| 4. When two angles and an included side | |

1. When three sides are given, as in Prob. XXIV. Case 5, viz.

AB

SPHER

FIG.28.

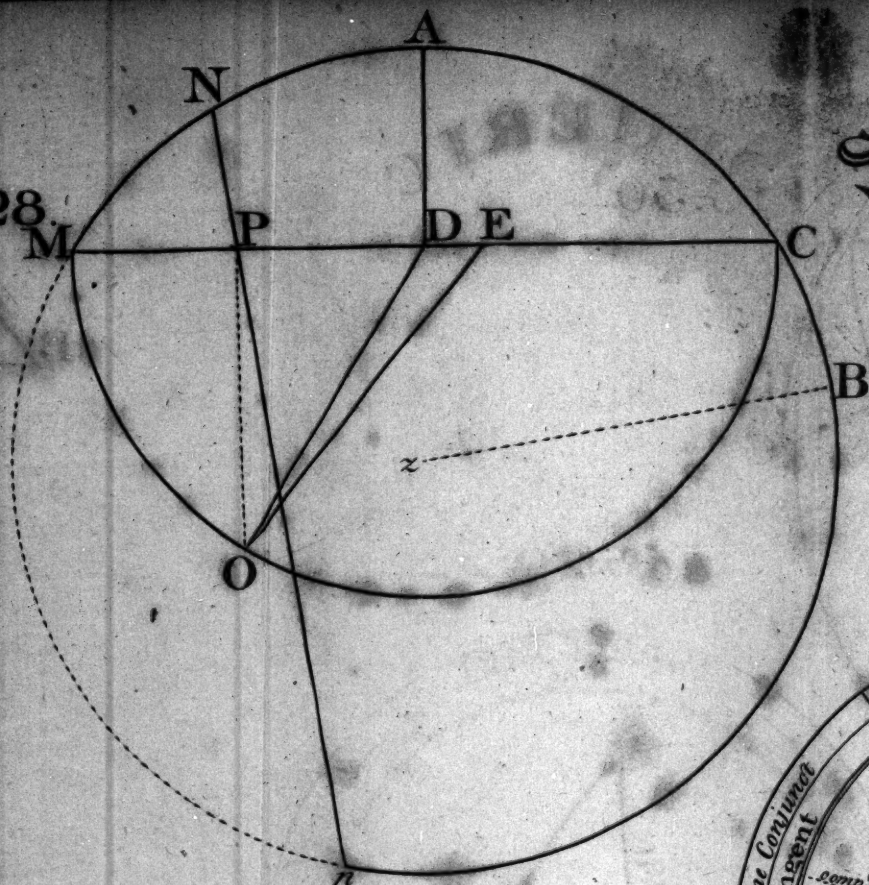


FIG 29

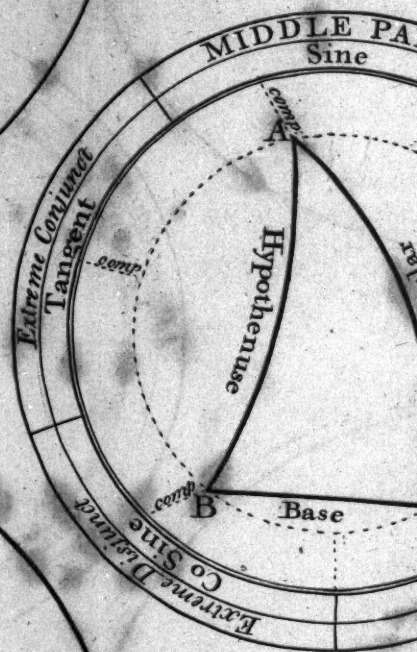
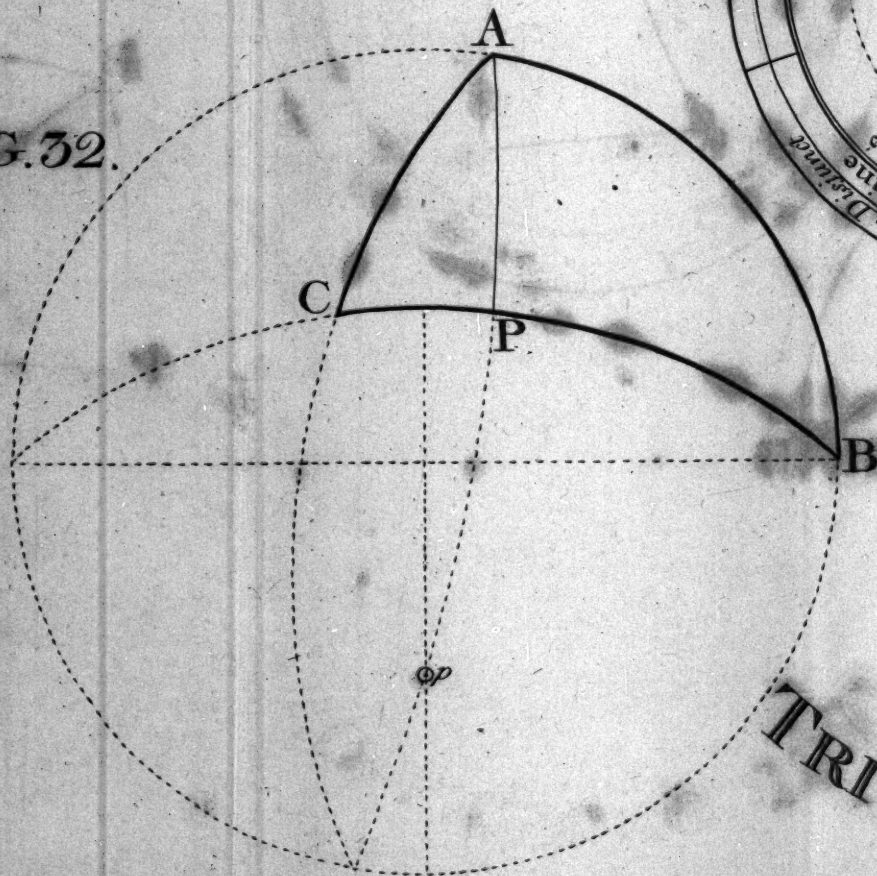


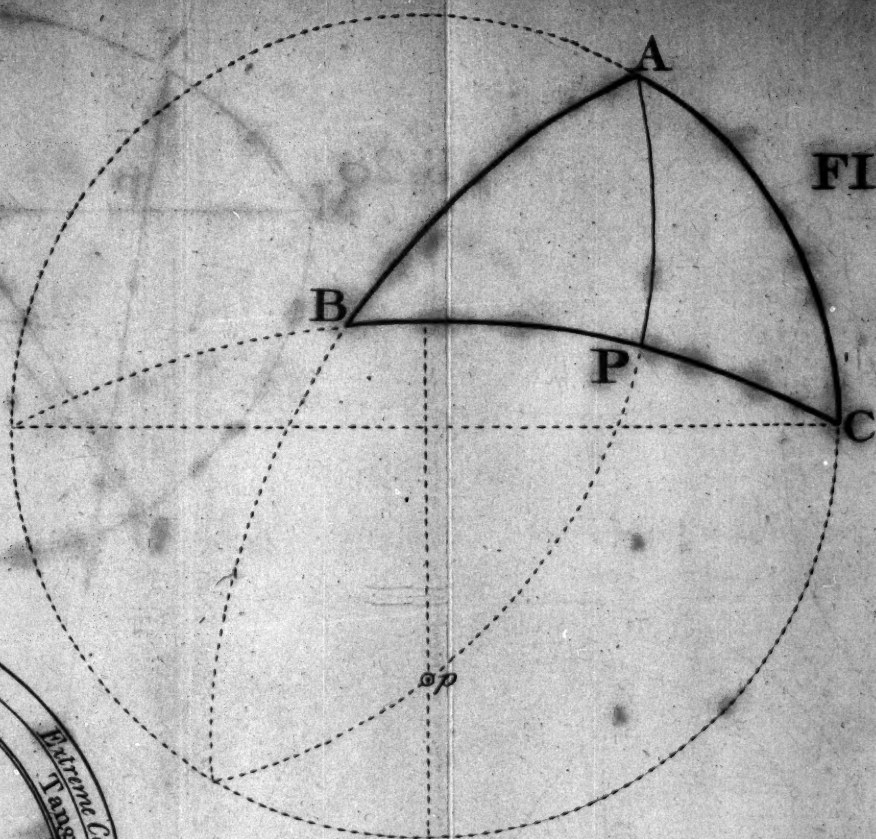
FIG.32.



TRIGONON

ERIC

FIG. 30.



G 29

LE PART
Sine

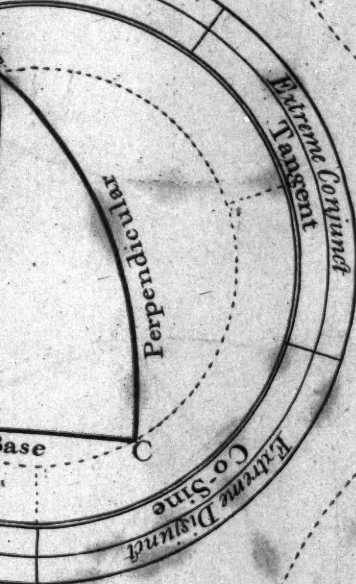
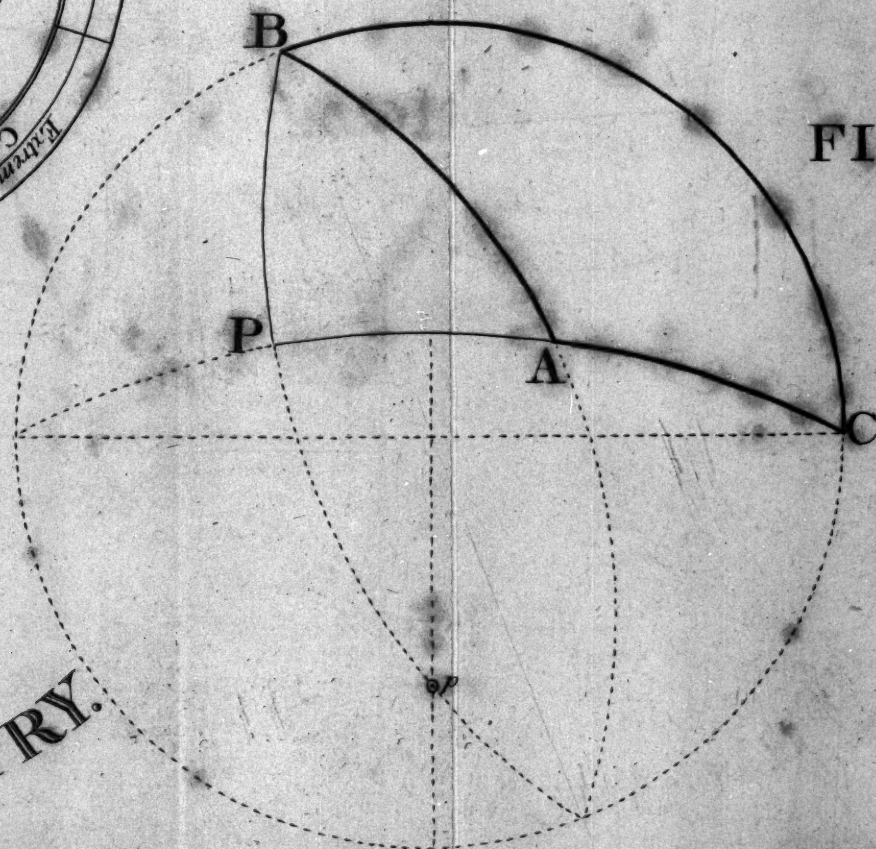


FIG. 31.



NOMETRY.

$$\left. \begin{array}{l} AB = 79^{\circ} 17' \\ BC = 110^{\circ} \\ AC = 58^{\circ} \end{array} \right\} \text{to find an angle, suppose } A.$$

With the sweep of 60 describe the circle $AMnBC$; from any point, as A , lay off the two sides AC and AB , which contain the required angle.

On the other side of the circle lay off $AM =$ to either AC or AB , suppose equal to AC , and draw the chord MC .

Then from the point B lay off the third side BC both ways, which will extend to n and N , and draw the chord Nn , which will cut MC in P .

With half MC , round the point D , describe the semi-circle MOC ,

From the point P erect a perpendicular PO , and draw the radius OD .

Then is the angle $ODC = 121^{\circ} 55' =$ the angle (A) required.

To find the angle B.

From O to MC lay off half the chord Nn , which will extend to E .

Then is the angle OED equal $50^{\circ} =$ the angle (B) required.

To find the angle C.

In this case it will be necessary to make a new construction, and the angle C may be known in the same manner that the angle A was found.

II. *When the three angles are given, the sides are found by the same construction, as Fig. 28.—That is,*

By using the supplements of the angles, as sides, and seeking a contained side, as the contained angle A was sought in the foregoing operation, and the supplement of the angle thus found will be the required side.

III. *When two sides and an included angle are given, lay off the two legs A B and A C, and draw M C as before.*

With the radius D C describe M O C.

Make the angle O D C equal to the given angle.

From O let fall a perpendicular to the line M C, which will cut it in P.

Through the point P draw the chord N n, perpendicular to B z, (continued) and B N or B n is the side required.

Again, take O E equal to half N n, and O E D equals the angle opp. the side A C.

IV. *When two angles and an included side are given, the third angle and either of the two sides are found by taking the supplements of the given angles and side, and proceeding*
as

as in the last instance; then the supplement of the side which is found, is the angle sought, and the supplement of the angle found is the side sought.

This method of solving Spheric Triangles is not numbered as the other articles, no reference being made to it throughout the work; it will however be found very simple and correct, as far as it extends: but projecting triangles, as in Section 12 and 13, should be preferred being the most natural and universal method, and affording most light and instruction, particularly where any cases of ambiguity occur.

The following solutions (being determined to seconds,) vary a little from the projections, for no instruments can attain the exactness of calculation.

SECTION XVI.

FIG. 29.

SPHERIC TRIGONOMETRY.

CXXXVIII. SPHERIC Trigonometry teaches to compute the sides and angles of spheric triangles from certain proportions between the sines, tangents, &c. of the parts that are known and those that are unknown.

CXXXIX. Every spheric triangle being composed of six parts, namely, three sides and three angles, when any three are given, the rest may be thence found; but in a right angled spheric triangle it is sufficient that two only be given, the right angle being always known.

CXL. Spheric Trigonometry is generally divided into three parts; viz. right angled, oblique angled, and quadrantal, (xxxiv.) and the two latter are founded upon the former.

CXLI. *Right angled spheric trigonometry* resolves itself into the six following cases:

- | | | |
|----|-------------------------------|--------------|
| 1. | When the hypotenuse and a leg | } are given. |
| 2. | the hypotenuse and an angle | |
| 3. | a leg and its opposite angle | |
| 4. | a leg and its adjacent angle | |
| 5. | two legs | |
| 6. | two angles | |

The

The foregoing six cases branch into sixteen analogies, which are founded on the following principles.

CXLII. In every right angled spheric triangle, the sines of the sides are in proportion to the sines of the opposite angles.—That is,

As the sine of any given side
Is to the sine of its opposite angle,
So is the sine of any other given side
To the sine of its opposite angle.

And,

As the sine of any given angle
Is to the sine of its opposite side,
So is the sine of any other given angle
To the sine of its opposite side.

CXLIII. But as certain cases occur in right angled spheric trigonometry, where a side and its opposite angle are not both given, recourse must be had to other proportions.

The following general rule, ascribed to Lord Napier, equally applies to all cases of right angled spheric triangles; it is called *The five circular parts*, by some *The catholic proposition*, and is deservedly adopted by almost every writer on the subject.

Of the five circular parts. FIG. 29.

CXLIV. The circular parts of a right angled spheric triangle are five, namely, the two legs, the complement of the hypotenuse and the complements of the two angles, (the right angle being always omitted.)

CXLV. Three

CXLV. Three of these circular parts, beside radius, enter every proportion, two of which are given, and the third sought.

CXLVI. These three parts are named from their positions with respect to one another, that is, according as they are joined or disjoined, observing that the right angle does not separate the legs.

CXLVII. If the three circular parts join, that which is in the middle is called the *middle part*, and the other two are called *extremes conjunct*.

CXLVIII. If the three circular parts do not join, two out of the five must, and that part which is separate or alone is the *middle part*, and the other two are called *extremes disjunct*.

These things being understood, the following is the general rule.

CXLIX. The product of radius, and the sine of the middle part, is equal to the product of the tangents of the extremes conjunct.

CL. And the product of radius, and the sine of the middle part, is equal to the product of the cosines of the extremes disjunct.

From these two equations analogies may be easily formed, observing always to take the complements of the angles and hypotenuse; and that the cosine of a complement is a sine, and the tangent of a complement is a co-tangent, and vice versa.

This

This noble proposition may be considered the foundation of Spheric Trigonometry : its principal excellence consists in the help which it affords to the memory ; and in order to promote this object still more, it is here put into verse. The many uncouth terms which necessarily occur, and indeed the subject itself are equally ill adapted for poetry : the verse however, such as it is, will be found useful, as connected and compared with the prose ; the one will aid the memory, and the other the understanding.

Each right angled spheric triangle contains
Five circular parts, as Lord Napier explains :
Three sides and *two angles* the circle surround,
 'Mongst which the *right angle* must never be found ;
 And instead of the *angles* and *hypothenuse*,
Their difference to ninety, or *complements*, use.
 With *radius*, *three parts* each analogy claims,
 Their relative places establish their names,
 Thus one's called the *middle*, the others *extremes*.
 If the parts be *connected*, the *extremes* are *conjunct*,
 But if *not connected*, then call them *disjunct*.

Now the *product* of *radius*, and *middle parts sine*,
 Equals that of the *tangents* of parts that *combine*,
 And also the *cosines* of those that *disjoin*.

SECTION XVII.

TO TURN EQUATIONS INTO ANALOGIES, AND TO VARY THE PROPORTIONS.

CLI. BY Euclid, B. 6, and P. 16, it is demonstrated that if four quantities be proportional, the product of the means equals the product of the extremes; thus, As 2 is to 4, so is 8 to 16, then will $2 \times 16 = 4 \times 8$.

CLII. To resolve the above equation back to an analogy, let the two numbers on either side be made the two means or the two extremes, then $2 \times 16 = 4 \times 8$ may be turned into the following analogies,

$$\begin{array}{l} 2 : 4 :: 8 : 16 \\ 2 : 8 :: 4 : 16 \\ 4 : 2 :: 16 : 8 \\ 4 : 16 :: 2 : 8 \end{array}$$

And these may be further varied by alternation, inversion, composition, division, conversion, &c. 5 Euclid, def. 12 to 16.

CLIII. When the equation contains an unknown quantity, this should be put last in the analogy, and the number connected with it must of course be put first; thus if $2 \times 16 = 8 \times x$, and x be required, say, as $8 : 2 :: 16 : x$, then $\frac{2 \times 16}{8} = x = 4$.

CLIV. The

CLIV. The proportions between the fines, tangents, and secants, may be further varied from the nature of similar triangles, thus,

As the co-fine is to the fine, so is radius to the tangent.

As the co-fine is to radius, so is radius to the secant.

As the fine is to radius, so is radius to the co-secant.

As the fine is to the co-fine, so is radius to the co-tangent.

CLV. There are, beside these, several variations arising from other relations between the fines, tangents, &c. and their comp. and these may be still further changed by Art. CLII. The following solutions are taken simply as they arise from the circular parts, but the various proportions which arise from combination, substitution, &c. will be given in Section 22.

SECTION XVIII.

FIG. 29.

THE NUMERICAL SOLUTION OF RIGHT ANGLED SPHERIC TRIANGLES.

Case 1. Given the Hypothenuse $AB = 63^{\circ} 56' 7''$, and a Leg $BC = 40^{\circ}$, to find the rest.

CLVI. To find the other leg:—

Here

Here the Hypotenuse and the two legs are the three circular parts which enter the proportion (cxlv).

The hypotenuse being disjoined from the legs by the angles, is therefore the middle part; and the legs are extremes disjunct (cxlviii); then $\text{Rad.} \times \text{Cof. Hyp.} = \text{Cof. Leg} \times \text{Cof. Leg}$ (cl); and by Art. cxi.

As Cof. given Leg B C	40°	Co-arc	0,1157460*
Is to Rad.	90°		10
So is the Cof. Hyp.	$63^\circ 56' 7''$		9,6428464
To Cof. required Leg A C	55°		9,7585924

The leg A C is acute, because the hypotenuse and given leg are of the same affection (xxx i).

The three sides and the right angle being known, the other angles may be determined by Art. cxlii, but if either angle were first required, the process must be as in the two following articles:

* The Co-arc or Arithmetical Comp. is found by subtracting the given arc from 10; or if it be more than 10, from 20: this may be done by beginning at the left or first figure, and subtracting each from 9, and the last from 10; thus the Cof. 40 degrees is 9,8842540; each of these subtracted from 9, except the 4, and this taken from 10, will leave 0,1157460, the Co-arc of Cof. 40 degrees.

The use of the Co-arc is to perform subtraction by addition, it is founded upon the common rule in subtraction of borrowing and restoring; thus, to subtract 7 from 9; the Co-arc of 7 is 3, which added to 9, rejecting the 10 that was borrowed, gives 2.

By the Co-arc and the help of Logarithms, subtraction, multiplication, and division are all performed by addition.

CLVII. *To find the angle B.*

Here this angle connects the hypotenuse and the given leg, and is therefore the middle part, and the other two extremes conjunct (CXLVII.)

Then $\text{Rad.} \times \text{Cof. } \angle B = \text{Tan. } LBC \times \text{Cot. Hyp. } AB$ (CXLIX), and by Art. CLIII,

As Radius 90°	Arith. Comp.
Is to Tan. $BC\ 40^\circ$	9,9238135
So is Cot. $AB\ 63^\circ\ 56'\ 7''$	9,6894258
To Cof. $B\ 65^\circ\ 46'\ 5''$	9,6132393

The angle B is acute, as the hyp. and given leg are of the same affection (XXXI).

CLVIII. *To find the angle A.*

Here the leg BC is the middle part, and the hyp. and required angle A extremes disjunct (CXLVIII); then $\text{Rad.} \times \text{Sine } BC = \text{Sine Hyp.} \times \text{Sine } A$ (EL); and by Art. CLIII,

As Sine Hyp. $AB\ 63^\circ\ 56'\ 7''$	0,0465794
Is to Radius 90°	10
So is Sine $BC\ 40^\circ$	9,8080675
To Sine A $45^\circ\ 41'\ 21''$	9,8546469

The

The angle A is acute, the hypotenuse and given leg being of the same affection.

C A S E 2.

Given $\left\{ \begin{array}{l} \text{the Hypotenuse AB } 63^{\circ} 56' 7'' \\ \text{and either Ang. sup. A } 45^{\circ} 41' 21'' \end{array} \right\}$ to find the rest.

CLIX. *To find the angle B.*

Here the hyp. is the middle part, and the angles extremes conjunct (cxlvii); then Rad. $\times$ Cos. Hyp. = Cot. A $\times$ Cot. B; and by Art. cliii,

As Cot. A	$45^{\circ} 41' 21''$	0,0104486
Is to Radius	90°	10
So is Cos. AB	$63^{\circ} 56' 7''$	9,6428464
To Cot. B	$65^{\circ} 46' 5'$	9,6532950

The angle B is acute, because the hyp. and given leg are of like affection (xxx1.)

CLX. *To find the adjacent leg AC.*

Here the angle A is the middle part, and the leg AC, and the hyp. AB extremes conjunct (cxlvii).

Then Rad. $\times$ Sine BC = Sine A $\times$ Sine AB (cl), and by Art. cliii.

As Radius	90°	Arith. Comp.
Is to Sine AB	$63^{\circ} 56' 7''$	9,9534206
So is Sine BC	$45^{\circ} 41' 21''$	9,8546469
To Sine BC	40°	9,8080675

The

SPHERIC TRIGONOMETRY.

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The side BC is acute, the hypotenuse and given angle being of the same affection.

C A S E 3.

Given a leg, supp. $AC = 55^\circ$
And its opp. angle $B = 65^\circ 46' 5''$ } to find the rest.

CLXII. *To find the other angle A.*

Here B is the middle part, and AC and A extremes disjunct (CXLVIII).

Then radius $\times$ Cos. B = Cos. AC $\times$ sine A (CL), and (by CLIII),

As Cos. AC 55°	0,2414087
Is to Radius 90°	10
So is Cos. B $65^\circ 46' 5''$	9,6132407
To sine A $45^\circ 41' 21''$	<u>9,8546494</u>

The angle A is here ambiguous or doubtful, as it cannot be determined by the data alone, whether BC, A and AB are greater or less than 90° each, but the construction (CXXVI and CXXVII) shews that they are each less than 90° .

CLXIII. *To find the other leg BC.*

Here BC is the middle part, and AC and B extremes conjunct (CXLVII).

F

Then

Then radius $\times$ sine $BC = \tan. AC \times \cot. B$ (cxl ix),
and (by cl iii),

As Radius	10°	
Is to Tan. AC	55°	10,1547732
So is Cot. B	$65^{\circ} 46' 5''$	9,6532976
To Sine BC	40°	9,8080708+

The side BC is doubtful for the same reasons that A is doubtful in the last article.

CLXIV. To find the Hypotenuse AB .

Here the leg AC is the middle part, and AB and B extremes disjunct (cxl viii).

Then radius $\times$ sine $AC = \text{fine } AB \times \text{fine } B$ (cl), and
(by cl iii),

As Sine B	$65^{\circ} 46' 5''$	0,0400568
Is to fine AC	55°	9,9133645
So is Radius	90°	10
To Hyp. AB	$63^{\circ} 56' 7''$	9,9534213+

The hyp. AB is doubtful for the same reason as in art. (clxii).

C A S E 4.

Given a Leg supp. $AC = 55^{\circ}$ } to find the
And its adj. Angle $A = 45^{\circ} 41' 21''$ } rest
CLXV To.

CLXV. *To find the other leg B C.*

Here A C is the middle part, and B C and A extremes conjunct (CXLVII).

Then radius $\times$ fine A C = cot. A $\times$ tan. B C (CXLIX), and (by CLIII),

As Cot. A	45° 41' 21"	0,0104486
Is to Radius	90°	10
So is fine A C	55°	9,9133645
To Tan. B C	40°	9,9238131 —

The side B C is of the same affection as the given angle A (XXXIX).

CLXVI. *To find the other angle B.*

Here the angle B is the middle part, and A C and A extremes disjunct (CXLVIII).

Then radius $\times$ cos. B = Cos. A C $\times$ fine A (CL), and (by CLIII),

As Radius	90°	
Is to Sine A	45° 41' 21"	9,8546465
So is Cos. A C	55	9,7585913
To Cos. B	65 46 5	9,6132388

The angle B is like the given leg A C (XXXIX).

CLXVII. *To find the Hypothenufe.*

Here the angle A is the middle part, and A C and A B extremes conjunct (cxlvi).

Then radius $\times$ Cos. A = tan. A C $\times$ cot. A B (cxlix);
and (by clii),

As Tan. A C	55°	9,8452268
Is to Radius	90°	10
So is Cos. A	45° 41' 21"	9,8441979
To Cot. A B	63 56 7	<u>9,6854247 —</u>

The hypothenufe AB is acute, as the given leg and angle are of the same affection.

C A S E 5.

Given the two legs $\begin{cases} AC = 55^\circ \\ BC = 40^\circ \end{cases}$ to find the rest.

CLXVIII. *To find the Hypothenufe.*

Here the hyp. is the middle part, and the legs extremes disjunct (cxlviii).

Then radius $\times$ cos. hyp. = cos. leg $\times$ cos. leg (cl),
and (by clii),

As

SPHERIC TRIGONOMETRY.

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As Radius 90°

Is to Cos. $AC = 55^\circ$

9,7585913

So is Cos. $BC = 40^\circ$

9,8842540

To Cos. AB

$63^\circ 56' 7''$

9,6428453—

The *hyp.* is acute, as both legs are of like affection.

CLXIX. To find an angle, suppose A .

Here the next side to the required angle is the middle part, that is, AC is the middle part, and A and BC extremes disjunct (CXLVII).

Then radius $\times$ sine $AC = \tan, BC \times \cot. A$ (CXLIX), and (by CLIII),

As Tan. BC

40°

0,0761865

Is to Radius

90°

10

So is Sine AC

55°

9,133645

To Cot. A

$45^\circ 41' 21''$

9,9895510—

The angle is acute like the *opp.* leg.

C A S E 6.

Given the two angles $\left\{ \begin{array}{l} A = 45^\circ 41' 21'' \\ B = 65^\circ 46' 5'' \end{array} \right\}$ to find the rest.

F 3

CLXX. To

CLXX. *To find the Hypotenuse.*

Here the hyp. is the middle part, and the angles extremes conjunct (cxlvi).

Then radius $\times$ cos. hyp. = cot. angle A $\times$ cot. angle B (cxlix), and (by cliii),

As Radius	90°	
Is to Cot. A	45° 41' 21"	9,9895514
So is Cot. B	65 46 5	9,6532976
To Cos. A B	63 56 7	9,6428490

The hyp. A B is acute, as the angles are of the same affection.

CLXXI. *To find either leg, suppose A C.*

Here the angle B opposite the required leg is the middle part, and the other angle A, and the leg A C extremes disjunct (cxlviii).

Then radius $\times$ cos. B = sine A $\times$ cos. A C (cl), and by Art. cliii,

As Sine A	45° 41' 21"	0,1453535
Is to Radius	90	10
So is Cos. B	65 46 5	9,6132407
To Cos. A C	55	9,7585942 +

SECTION

SECTION XIX.

OF THE SOLUTION OF QUADRANTAL TRIANGLES.

QUADRANTAL Triangles are solved upon the same principles as Right Angled Spheric Triangles, observing that the quadrantal side be called radius, and the supplement to the angle opposite that side be called the hyp. and that the other sides be called angles, and their opposite angles, legs:—And further observing that where the kind of a side or angle is determined by the hyp., or the hyp. is to be determined, to use the word unlike instead of like, and like instead of unlike.

Thus, if we suppose a quadrantal triangle whose hypotenusal angle equals $116^{\circ} 3' 53''$, (the supp. $63^{\circ} 56' 7''$) and the other two angles equal 55° and 40° ; then will the legs = $65^{\circ} 46' 5''$ and $45^{\circ} 41' 21''$, and the various solutions of such a triangle will be the same as the foregoing solutions of right angled triangles. Examples of quadrantal triangles are therefore deemed unnecessary in this place, especially as such operations rarely occur in nautical practice.

SECTION XX.

THE NUMERICAL SOLUTION OF OBLIQUE SPHERIC TRIANGLES.

EVERY Oblique Spheric Triangle being composed of six parts, namely, three sides and three angles, any three being given, the rest may be found.

The solution of oblique spheric triangles, therefore, admits of six cases, viz.—When

- | | |
|------------------------------------|--------------|
| 1. Two sides and an opp. angle | } are given. |
| 2. Two sides and an included angle | |
| 3. Two angles and an opp. side | |
| 4. Two angles and an included side | |
| 5. Three sides | |
| 6. Three angles | |

CLXXII. In every oblique spheric triangle as in a right one (CXLII). *The sines of the sides are in proportion to the sines of their opposite angles.*

This rule should be preferred for its simplicity whenever it can be applied; but in most cases recourse must be had to other proportions.

The most approved method of solving any of the above four first cases is by means of a perpendicular let fall from an angle to its opp. side (continued if necessary);

fary); thus two right angled triangles are formed, which may be solved by the rules already given in Right Angled Spheric Trigonometry.

But in order to let fall the perpendicular so as to have sufficient data in one of the new triangles, the following rule must be observed,

CLXXIII. *Let the perpendicular be so drawn that two of the given things in the oblique triangle may remain known in one of the right angled triangles.*

Now in any of the four first cases, a side and its adjacent angle being given, let this side be laid off on the primitive, as AC Fig. 30, and let the given adjacent angle be laid off at C (cix); then from the angle A draw a great circle perpendicular to BC (cxii); and AP will be the perpendicular required.

CLXXIV. When the angles next the side, upon which the perpendicular should fall, are of the same affection, the perpendicular will fall within the triangle as Fig. 30 and 32; but when those angles are unlike, as Fig. 31, the perpendicular will fall without the triangle, and thus join a right angled triangle BPA to the given oblique triangle.

CLXXV. When the perpendicular is let fall, the segments of the base and their opp. angles are thus found (Fig. xxx).

$$\left. \begin{array}{l} \text{As Cot. } AC : \text{Rad.} :: \text{Cos. } C : \text{Tan. } PC \\ \text{As Cot. } AB : \text{Rad.} :: \text{Cos. } B : \text{Tan. } PB \end{array} \right\} (\text{CLX}).$$

And

$$\left. \begin{array}{l} \text{As Cot. } C : \text{Rad.} :: \text{Cos. } AC : \text{Cot. } PAC \\ \text{As Cot. } B : \text{Rad.} :: \text{Cos. } AB : \text{Cot. } BAP \end{array} \right\} (\text{CLIX}).$$

The sum or difference of the segments or angles must be taken according as the perpendicular falls, within or without the triangle.

CLXXVI. The following proportion between the angles at the base and the segments of the base will be found useful in certain cases.

The product of the sine of either segment, and the tangent of its adjacent angle, equals the product of the sine of the other segment and the tangent of its adjacent angle.

Thus Fig. 30, $\text{fine } BP \times \tan. B = \text{fine } PC \times \tan. C$; hence any three being given, the fourth may be found (CLIII).

C A S E 1.

CLXXVII. Given two sides and an opp. angle, viz.

$$\left. \begin{array}{l} \text{Side } AC = 58^{\circ} \\ \text{Side } AB = 79^{\circ} 17' 14'' \\ \text{Angle } C = 62^{\circ} 34' 6'' \end{array} \right\} \text{to find the rest.}$$

Let the triangle be projected (CXXXI), and the perpendicular let fall (CLXXIII).

Then

Then to find the angle B , (CLXXII).

As the Sine Side AB	$79^{\circ} 17' 14''$	0,0076359
Is to Sine $\angle C$	$62^{\circ} 34' 6''$	9,9481982
So is Sine Side AC	58	9,9284205
To Sine $\angle B$	50	9,8842546 +

The angle B may be either acute or obtuse, from the things given, but the construction shews it to be acute.

Now the side BC , and its opp. angle A , are still unknown, and either being found (by Art. CLXXV), the other is determined by Art. CLXXII.

Thus, to find the segments BP and PC .

As cot. side AC	58°	0,2042108	As cot. side AB	$79^{\circ} 17' 14''$	0,7231180
Is to radius	90°	10	Is to radius	90°	10
So is cos. $\angle C$	$62^{\circ} 34' 6''$	9,6634092	So is cos. $\angle B$	50	9,8080675
To tan. seg. PC	$36 24$	9,8676200	To tan. seg. BP	$73 36$	10,5311855
PC is acute, the hyp. AC and the angle C being alike.			PB is acute, the hyp. AB and angle B being alike.		

The angle B and the angle C being alike, the perpendicular will fall within the triangle (CLXXIV); and therefore the segments BP and PC must be added together, which gives the side $BC = 110^{\circ}$.

Then

Then to find the angle A (CLXXII).

As Sine Side AC	58°	0,0715795
Is to Sine ∠ B	50	9,8842540
So is Sine Side BC	110	9,9729858
To Angle A	121° 54' 56"	9,9288193

The angle A may be either obtuse or acute from the things given, but the construction shews it to be obtuse.

C A S E 2.—FIG. 30.

CLXXVIII. Given two sides and an included angle, viz,

The Side AC	=	58°	} to find the rest.
The Side BC	=	110°	
The included ∠ C	=	62° 34' 6"	

The triangle being projected (CXXXIII), and a perpendicular let fall (CLXXIII).

To find the segment PC (CLX),

As Cot. Side AC	58°	} See the second analogy of the last case.
Is to Radius	90	
So is Cos. ∠ C	62° 34' 6"	
To Tan. PC	36 24	
		110° - 36° 24' =
		73° 36' = B P.

To

To find the angle *B* (CLXXVI).

As Sine Seg. B P	73° 36'	0,0180392	} The angle B may be also found by saying as radius :
Is to Sine P C	36 24	9,7733614	
So is Tan. ∠ C	62° 34' 6"	10,2847890	
To Tan. ∠ B	50	10,0761896 +	
fine B C :: fine C to perpendicular A P. And then			

As tan. *AP* : radius :: fine *BP* : cot. *B* (CLXIX).

To find the angle *A*. (CLXXII).

As Side <i>AC</i>	58°	} as in the last analogy of the last case.
Is to Angle <i>B</i>	50°	
So is Side <i>BC</i>	110°	
To Angle <i>A</i>	$121^{\circ} 54' 6''$	

CLXXIX. If in this case the side 110 had been put upon the primitive, the construction would have been as in Fig. 31, and the solution as follows.

To find the leg *PA*, let the whole leg *PB* be first sought, (CLX).

As Cot. BC	110°	0,4389341	} The side BP is obtuse, ashyp. and the angle C are unlike.
Is to Rad.	90°	10	
So is Cos. C	62° 34' 6"	9,6634092	
To Tan. PC	128 18 38	10,1023433	
PC — Ac = 70° 18' 38" = the seg. PA.			

To

To find the Perp. (CLXI). To find BA (CLXVIII).

As rad. 90°
Sine AB 110° 9,9729858
So is sine $\angle C$ $62^\circ 34' 6''$ 9,9481982

To sine AP $56^\circ 30' 56''$ 9,9211840—

AP is acute like its opp. angle C.

As rad. 90°
Cos. PA $70^\circ 18' 38''$ 9,5275291
Cos. PB $56^\circ 30' 56''$ 9,7417113

Cos. BA $79^\circ 17' 14''$ 9,2692404—

The hyp. BA is acute, because the legs are alike.

To find the angle A, (CLXXII). To find the angle B, (CLXXII).

As sine side AB $79^\circ 17' 14''$ 0,0076359
Is to sine $\angle C$ $62^\circ 34' 6''$ 9,9481982
So is sine side BA 110° 9,9729858

To sine $\angle A$ $121^\circ 54' 56''$ 9,9288199+

The angle A may be acute or obtuse from the things given, but the construction shews it to be obtuse.

As sine side CB $79^\circ 17' 14''$ 0,0076359
Is to sine $\angle C$ $62^\circ 34' 6''$ 9,9481982
So is sine AC 58° 9,9284205

To sine $\angle B$ 50° 9,8842546+

The angle B may be acute or obtuse from the things given, but the construction shews it to be acute.

This case may be performed more concisely, thus:—
when the segments PA and PC are found by the first analogy;

Find the angle BAP, the supplement of the angle BAC. (CLXXVI.)

As Sine Leg PA $70^\circ 18' 38''$ 0,0261646
Is to Sine Leg PC $128^\circ 18' 38''$ 9,8946827
So is Tang. Ang. C $62^\circ 34' 6''$ 10,2847890

To Tang. Ang. BAP $58^\circ 5' 4''$ 10,2056363

Subtract $58^\circ 5' 4''$ from 180° , the rem. is $121^\circ 54' 56''$, the angle BAC; this being known, the rest may be found by Art. CLXXII.

C A S E

C A S E III.—FIG. 32.

Given two angles and an opposite side, viz.

The angle B = 50°

The angle C = $62^{\circ} 34' 6''$

The side AB = $79^{\circ} 17' 14''$

The triangle being constructed, (cxxxiv) and the perpendicular let fall (clxxiii),

To find the side CA. (clxxii.)

As the Sine $\angle C$ $62^{\circ} 34' 6''$	00518018	} The Side CA may be acute or obtuse from the data, but the construction shews it to be acute.
Is to Side AB $79^{\circ} 17' 14''$	99923641	
So is Sine $\angle B$ 50°	9,8842540	
To Sine Side CA 58	9,9284199	

Here the angle A and its opposite side are still unknown, and either being found, the other is determined by Art. clxxii.

To find the segment PA. (clx.)

As Cot. AC $79^{\circ} 17' 14''$	0,7231180	} PB is acute, the hyp. AB and the angle B being alike.
Is to Radius 90°	10	
So is Cos. $\angle B$ 50°	9,8080675	
To Tan. BP $73^{\circ} 36'$	10,5311855	

To

To find the other segment. (CLXXVI.)

As Tan. $\angle C$	$62^{\circ} 34' 6''$	9,7152110	} C P is acute; the Hyp. C A being acute.
Is to Tan. $\angle B$	50	10,0761865	
So is Sine Seg. P B	73 36	9,9819608	
To Sine Seg. P C	36 24	9,7733583	

The sum of the segments P B and P C = 110, the side C B.

To find the angle A. (CLXXII.)

As fine 58°	} See last analogy of Case I. (CXXVII.)
Is to fine 50°	
So is fine 110°	
To fine $121^{\circ} 54' 56'' =$	

the angle A, which may be acute or obtuse from the data, but the construction shews it to be obtuse.

C A S E IV.—FIG. 31.

Given two angles and an included side, viz.

The angle B	=	50°	} to find the rest.
The angle C	=	$62^{\circ} 34' 6''$	
The included side B	=	110	

The triangle being projected (CXXXV), and the perp. let fall (CXXXIII.)

To find the side *AB*.

Let the angle *PBC* be first found, (CLIX), then is the angle *PBA* known; next find the perp. *BP* (CLXI), and then *BA* (CLXVII), thus:

To find the angle *PB* (CLIX).

As Cot. $\angle C$	62° 34' 6"	0,2847890	} The angle <i>PBC</i> is obtuse, as the Hyp. <i>BC</i> and $\angle C$ are unlike.
Is to Rad.	90	10	
So is Cos. <i>BC</i>	110	95340517	
To Cot. <i>PBC</i>	123° 22' 56"	9,8188407	

Subtract 50° from 123° 22' 56", and there remains 73° 22' 56", the angle *PBA*.

To find the perp. *PB* (CLXI).

This is found in Analogy 3d, Art. 179, and equals 56° 30' 56".

From *PB* and *PBA* find *BA* (CLXVII).

As Tan. <i>PB</i>	56° 30' 56"	9,8205267
Is to Rad.	90	10
So is Cos. <i>PBA</i>	73 22 56	9,4563443

To Cot. <i>BA</i>	79 17 14	9,2768710-
-------------------	----------	------------

BA is acute, the side *BP* and $\angle PBA$ being alike, and being determined, the rest may be found as before, (CLXXII.)

C A S E V.—FIG. 30.

Given the three sides, viz.

<i>AB</i>	=	79° 17' 14"	} to find the rest.
<i>BC</i>	=	110	
<i>AC</i>	=	58	

G

CLXXXII. To

CLXXXII. *To find the angle A.*

From half the sum of the three sides subtract each of the two sides which contain the required angle.

Add together,

The sines of these two remainders, and the co-arcs of the sines of the sides which contain the angle.

Half the sum of these four logarithms will give the sine of half the required angle. Thus:

$$\begin{array}{r}
 79^{\circ} 17' 14'' \\
 110 \\
 \hline
 58 \\
 2) 247 \ 17 \ 14 \\
 \hline
 123 \ 38 \ 37 = \frac{1}{2} \text{ sum of three sides.} \\
 79 \ 17 \ 14 \\
 \hline
 44 \ 21 \ 23 \text{ first remainder} \quad \text{Sine } 9,8445513 \\
 123 \ 38 \ 37 \\
 58 \\
 \hline
 65 \ 38 \ 37 \text{ second remainder} \quad \text{Sine } 9,9595173 \\
 \text{Co-arc Sine } 58^{\circ} \quad 0,0715795 \\
 \text{Co-arc Sine } 79^{\circ} 17' 14'' \quad 0,0076359 \\
 \hline
 2) 19,8832840 \\
 \hline
 \text{Sine } 60^{\circ} 57' 28'' = \frac{9,9416420 + \frac{1}{2}}{2} \text{ of a second nearly.} \\
 \hline
 121 \ 54 \ 56 \text{ equals the required } \angle A.
 \end{array}$$

By

SPHERIC TRIGONOMETRY:

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By a similar operation the angles B and C may be found; but when one angle is known, the other two are most readily determined by Art. 172.

CLXXXII. *Another method.*

Find the difference between half the sum of the three sides and the side opposite the required angle A.

Then add together;

The sine of the said half sum,

The sine of the said difference,

The co-arcs of the sines of the two containing sides.

Half the sum of these four logarithms will be the cosine of half the required angle.

Half the sum of the } 123° 38' 37" sine 9,9203841
three sides
Side opp. $\angle A$ 110

Difference 13 38 37 sine 9,3726945

Co-arc 58° 0,0715795

Co-arc 79° 17' 14" 0,0076359

2) 19,3722940

Cos. 60° 57' 28" 9,6861470 $\frac{1}{2}$
of a sec. nearly.

Angle A 121 54 56

C A S E VI.—FIG. 30.

CLXXXIII. The three angles given, viz.

Angle A = 121° 54' 56" } to find the
Angle B = 50 } rest.
Angle C = 62 34 6 }
G 2 To

To find the Side BC.

From half the sum of the three angles take each of the angles next the required side.

Add together,

The co-fines of these two remainders, and

The co-arcs of the fines of each of the adjoining angles.

Half the sum of these four logarithms will give the co-fine of half the required side. Thus:

$$\begin{array}{r}
 121^{\circ} 54' 56'' \\
 50 \\
 \hline
 62 \quad 34 \quad 6 \\
 2) 234 \quad 29 \quad 2 \\
 \hline
 117 \quad 14 \quad 31 \\
 62 \quad 34 \quad 6 \\
 \hline
 54 \quad 40 \quad 25 \text{ first rem.} \\
 67 \quad 14 \quad 31 \text{ second rem.} \\
 \text{Co-arc Sine } 50^{\circ} \quad 0,1157450 \\
 \text{Co-arc Sine } 62^{\circ} 34' 6'' \quad 0,0518018 \\
 \hline
 2) 19,5171821 \\
 \hline
 \text{Cofine } 55^{\circ} \quad 9,7585910 \\
 2 \text{ about } \frac{1}{15} \text{ of a sec. more.} \\
 \hline
 110^{\circ} = \text{the required side BC.}
 \end{array}$$

By the same method the other sides may be found; but one side being known (with the angles) the rest are most readily found by Art. 172.

CLXXXIII. An-

CLXXXIII. *Another method.*

Find the difference between half the sum of the three angles and the angle A opposite the required side B C.

Add together,

The cosine of the said half sum,

The cosine of the said difference,

The co-arcs of the other two angles B and C.

Half the sum of these four logarithms will be the sine of half the required side B C.

Half the sum of 3 angles $117^{\circ} 14' 31''$ Cos. 9,6606273

The $\angle$ A opp. req. side 121 54 56

Difference 4 40 25 Cos. 9,9985536

Co-arc $\angle$ B = 50 0,1157450

Co-arc $\angle$ C = 62 34 6 0,0518018

2)19,8267277

Sine 55°

2

9,91336385—
nearly $\frac{1}{3}$ of a sec. less.

The required Side B C 110

SECTION XXI.

TO SOLVE OBLIQUE SPHERIC TRIANGLES WITHOUT
A PERPENDICULAR.

ALTHOUGH solving the four first cases of Oblique Spheric Triangles by means of a perpendicular is preferred, as affording most practice in right-angled spheric trigonometry, and in projection, yet the following method of solving those cases without a perpendicular has its advantages also, and therefore ought not to be omitted.

These solutions depend upon the four following Propositions.

PROP. 1.

CLXXXIV. As the sine of half the sum of two sides*
Is to the sine of half their difference,
So is the co-tangent of half their contained angle
To the tangent of half the diff. of the other angles.

PROP. 2.

CLXXXV. As the co-sine of half the sum of the sides
Is to the co-sine of half their difference,
So is the co-tangent of half the contained angle
To the tangent of half the sum of the other two angles.

PROP.

* Observe, if the sum of the two sides should exceed a semicircle, subtract each from 180° , and proceed with those remainders as the sides; the result will be the supplement to 180 of the required angle.

P R O P. 3.

CLXXXVI. As the fine of half the sum of two angles
Is to the fine of half the difference,
So is the tangent of half the contained side
To the tangent of half the diff. of the other two sides.

P R O P. 4.

CLXXXVII. As the co-fine of half the sum of two
angles
Is to the co-fine of half their difference,
So is the tangent of half the included side
To the tangent of half the sum of the other two sides.

These four Propositions may seem only applicable to
Cases 2d and 4th, but they may be equally applied to
Cases 1st and 3d, by alternation. Thus, in Prop. 1,
when a side and its opposite angle are unknown :

CLXXXVIII. As the fine of half the diff. of two sides
Is to the fine of half their sum,
So is the tangent of half the diff. of the two angles
To the co-tangent of half the contained angle.

C A S E 1.—FIG. 30.

CLXXXIX. Given two sides and an opp. angle,
viz,

The Side AC	=	58°	} to find the rest.
The Side BC	=	110	
The angle B	=	50	

G 4

First,

First, let the angle A be found (CLXXII) = $121^{\circ} 54' 56''$; then

To find the angle C. (CLXXXVIII.)

110°		$121^{\circ} 54' 56''$	
<u>58</u>		<u>50</u>	
168	Sum of fides	171 54 56	Sum of angles
<u>52</u>	Diff. of fides	<u>71 54 56</u>	Diff. of angles
84 $\frac{1}{2}$	Sum of fides	85 57 28 $\frac{1}{2}$	Sum of angles
<u>26 $\frac{1}{2}$</u>	Diff. of fides	<u>35 57 28 $\frac{1}{2}$</u>	Diff of angles
As Sine	26°	0,3581580	
Is to Sine	84	9,9976143	
So is Tan.	35 57 28	9,8605879	
To Co-tang. $\frac{1}{2} \angle C$	$31^{\circ} 17' 3''$	10,2163602 +	
	<u>2</u>		
The $\angle C$	<u>62° 34' 6''</u>		

Here C being found, the opposite side AB may be determined (CLXXII) = $79^{\circ} 17' 14''$.

C A S E II.—FIG. 30.

CXC. Given two fides and an included angle, viz.

The Side AC	=	58°	} to find the reft.
The Side BC	=	110°	
The included $\angle C$	=	$62^{\circ} 34' 6''$	

To

To find the sum and difference of the other two angles *A* and *B* (CLXXXIV and CLXXXV).

As Sine half sum 2 sides	84°	0,0023857
Is to Sine half their diff.	26°	9,6418420
So is Cot. half cont. angle C	31° 17' 3"	10,2163600
To Tan. half diff. 2 angles	35 57 28	9,8605877—
	2	
Diff. angles	71 54 56	

Again,

As Cos. half sum 2 sides	84°	0,9807653
Is to Cos. half diff.	26°	9,9536602
So is Cot. half cont. angle C	31° 17' 3"	10,2163600
To Tan. half the sum of ∠s	85 57 28	11,1507855+
	2	
The sum of angles	171 54 56	

Here half the sum of the angles added to half the difference gives the greater angle; and half the difference subtracted from half the sum gives the lesser angle.

85° 57' 28"	= half sum
35 57 28	= half difference
121 54 56	= greater angle A
50	= lesser angle B.

Hence the side *AB* may be found (CLXXXII) = 79° 17' 14".

CXC.

CXC. As this Case occurs frequently in astronomical calculations, particularly in those of the lunar observations, a shorter method is here given for determining the third side.

To the fines of the two containing sides
Add double the fine of half the contained angle.

From half the sum of these three logarithms
Subtract the fine of half the diff. of the sides,
The remainder is the tang. of an arc.

Subtract the fine of this arc from the said half sum of the 3 logarithms,

The remainder is the fine of half the required side.

Thus,

Side AC	=	58°	Sine	9,9284205
Side B		110	Sine	9,9729858
$\frac{1}{2}$ incl. angle		31° 17' 3"	double the fine	19,4308082

2)39,3322145

$\frac{1}{2}$ diff. sides		26°	Sine	19,6661072
				9,6418420

Tangent		46° 35' 59"		10,0242652
---------	--	-------------	--	------------

Sine		46 35 59		9,8612783
------	--	----------	--	-----------

Subtracted from half Sum of log, 9,8048289

Sine 39° 38' 37"
2

The required side AB 79 17 14

CASE

C A S E III.—FIG. 30.

CXCI. Given two angles and an opp. side, viz.

The Angle A	=	121° 54' 56"	} to find the rest.
The Angle B	=	50	
The Side AC	=	58	

Let the side BC be first found (CLXXII) = 110°;
then,

To find the angle C (CLXXXVIII).

The operation here is the same as in Art. 189.

C A S E 4.—FIG. 30.

CXCII. Given two angles and an included side, viz.

The Angle A	=	121° 54' 56"	} to find the rest.
The Angle C	=	62 34 6	
The incl. side AC	=	58	

To find the sum and difference of the other two sides,
(CLXXXVI and CLXXXVII.)

121° 54' 56"	
62 34 6	
184 29 2	Sum of sides
59 20 50	Diff. of sides
92 14 31	Half sum of sides
29 40 25	Half diff. of sides.

As Sine	92° 14' 31	0,0003326
Is to Sine	29 40 25	9,6946566
So is Tang. half side	29	9,7437520
To Tan. half diff. other sides	15 21 23	9,4387502 +

Again:

Again:

As Co-sine	$92^{\circ} 14' 31''$	1,4076086
Is to Co-sine	$29^{\circ} 40' 25''$	19,9389496
So is Tan. half side	29	<u>9,7437520</u>
To Tan. half sum of the other two sides	$94^{\circ} 38' 37''$	<u>11,0903102</u>

This half sum of the two sides comes out $85^{\circ} 21' 23''$;
but the supplement of this must be taken.

Add the half sum to the half diff. for the greater side,
and subtract the half sum from the half diff. for the lesser
side: thus,

$$\begin{array}{r}
 94^{\circ} 38' 37'' \\
 15 \quad 21 \quad 23 \\
 \hline
 110
 \end{array}
 \quad \text{the greater side} = BC$$

And

$$\begin{array}{r}
 94^{\circ} 38' 37'' \\
 15 \quad 21 \quad 23 \\
 \hline
 79 \quad 17 \quad 14
 \end{array}
 \quad \text{the lesser side } AB.$$

SECTION

SECTION XXII.

IMPROVED SOLUTIONS OF CERTAIN CASES OF SPHERIC TRIANGLES.

IT may be observed that the foregoing calculations, though carried to a greater degree of nicety than any yet published, do not all perfectly agree, some varying nearly $\frac{1}{3}$ of a second. This small error arises from the fraction of a second being in some part or parts of the triangle, and from the want of tables calculated to a sufficient length to take proportional parts for such small fractions. All spheric triangles are liable to errors of this description, but as such can never exceed half a second, they are deemed objects of no consideration, even where the greatest accuracy is required.

But there is another source of error in spheric solutions, which in some instances is of material consequence, though scarcely adverted to by any writer on the subject. This arises from the size of an arc and the denomination by which it is solved; that is, whether a large arc or a small one be brought out a sine or a cosine.

By a large arc is understood one between 45° and 135° ; by a small arc one under 45° or above 135° : the former is sometimes called a *mean arc*, and the latter an *extreme arc*; but it is only when the arc is very near 90° or 0° , that it is liable to an error of any consequence.

These

These vague or inaccurate solutions arise from the unequal manner by which the sines increase from 0° to 90° ; for the sines of small arcs increase by large increments, and the sines of large arcs by small increments: the contrary holds with regard to their cosines. The manner by which the sines increase is perhaps best ascertained by laws of that converging series, which is obtained from the operation of calculating sines by fluxions. But those who do not understand this science may easily conceive, that as the sine of a very small arc and the arc itself are nearly parallel, every removal of the sine must make a sensible change in its length. Whereas the sine of a very large arc being nearly perpendicular to the arc, a small removal of the sine will scarcely add any thing to its length; and therefore the sines near 90° increase by very slow and almost imperceptible gradations, while the sines near 0° increase by very large increments. This is seen in a table of sines: the logarithmic difference is large between the seconds near 0° , but so very small near 90° , that the sines in one part run nearly 100 seconds, without any difference whatever.

Now it is evident that an arc coming out in this part of the table cannot be accurately determined, as there is no rule by which one second should be preferred to another, where there is a choice of so many. To avoid such a vague solution, the following rule must be observed—*not to bring out the result of a large or mean arc a sine, nor the result of a small or extreme arc a cosine.*

But any arc coming out either a tangent or cotangent is a good solution, as the logarithmic differences of these are never so small as to leave any doubt of the real
I
answer.

answer. Hence those methods of calculating the lunar observations where the true distance is brought out a tangent or cotangent should be preferred, as being the most universally accurate.

In order to obtain an accurate solution where any of the above vague answers are likely to occur, the denomination must be changed, by finding the relative proportions between the fines, tangents, &c. of an arc, and then in the place of a fine or cofine substituting its value, which is performed by a simple operation in algebra.

The following equations shew the chief proportions between the fines, tangents and secants, and their complements of any arc, suppose A.

$$1 \quad \text{Sine } A = \frac{\text{Tan. } A \times \text{Cos. } A}{\text{Rad.}} = \frac{\text{Cos. } A \times \text{Rad.}}{\text{Cot. } A.}$$

$$2 \quad \text{Cos. } A = \frac{\text{Sine } A \times \text{Rad.}}{\text{Tan. } A} = \frac{\text{Sine } A \times \text{Cot. } A}{\text{Rad.}}$$

$$3 \quad \text{Tan. } A = \frac{\text{Sine } A \times \text{Rad.}}{\text{Cos. } A} = \frac{\text{Sine } A \times \text{Sec. } A}{\text{Rad.}}$$

$$4 \quad \text{Cot. } A = \frac{\text{Cos. } A \times \text{Rad.}}{\text{Sine } A} = \frac{\text{R R}}{\text{Tan. } A}$$

$$5 \quad \text{Sec. } A = \frac{\text{R R}}{\text{Cos. } A} = \frac{\text{Tan. } A \times \text{Rad.}}{\text{Sine } A}$$

$$6 \quad \text{Cofec. } A = \frac{\text{R R}}{\text{Sine } A} = \frac{\text{Cot. } \frac{1}{2} A + \text{Tan. } \frac{1}{2} A}{2}$$

These equations may be further varied, but it is unnecessary, as the first and second are sufficient for forming those accurate solutions by logarithms which are given in the Table.

General

General Remarks on the Correction of proportional Errors.

THERE are, beside the foregoing corrections, others where an error is *known* in one part of the triangle, from whence its proportional effects on the other parts are to be ascertained. In some triangles an error in one part will affect *all* the rest; in other triangles *one* part, and sometimes *two* remain unchanged. Thus, in a right-angled plane triangle an error in the angle at the base will not change either the right angle or the base, but all the other parts will be affected: in this case two parts are said to be *constant* and four *variable*.

These proportional errors have been mostly determined by fluxions; but Mr. Cagnoli, who has gone more largely into this subject than perhaps any other writer, determines them by very simple operations, which he calls *Differential Analogies*, and to this elaborate work the reader is referred: it is however a subject rather of theory than practice; for it may be objected, that if the error be *already known* it can be allowed for in the first instance, which will obviate the necessity of any subsequent corrections. Thus, in taking an altitude, if the error in the observed angle be known, it may be corrected before any other calculation is made, and thus it cannot affect the height of the object, or pervade any other part of the operation.

But the theory of estimating proportional errors in triangles affords some practical inferences. Thus in the case of taking altitudes it is found, that when the observed angle is 45° an error in the observation affects the height of the object less than at any other altitude, and this in the ratio of *Radius to double the Sine of the observed Angle*; hence altitudes taken near the horizon or zenith are liable to greater errors than those taken at or near 45° : but this is a subject more properly belonging to the second part of this work.

PART THE SECOND.

NAUTICAL ASTRONOMY;

BEING THE
APPLICATION OF SPHERICS

TO

THOSE PROBLEMS OF ASTRONOMY WHICH ARE
MOST USEFUL AT SEA:

SUCH AS FINDING THE
AZIMUTH, AMPLITUDE, TIME, LATITUDE,
LONGITUDE, &c.

ALL OF WHICH ARE SOLVED BOTH BY
PROJECTION AND CALCULATION.

THE WHOLE CONCLUDING WITH
A COMPARATIVE VIEW OF THE MOST APPROVED
METHODS OF WORKING

THE LUNAR OBSERVATIONS.

ILLUSTRATED BY

FIGURES, SHEWING THE PRINCIPLES AND RATIONALE
UPON WHICH THOSE CALCULATIONS
ARE FOUNDED.

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THE SECOND

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ANALYSIS

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NAUTICAL ASTRONOMY.

SECTION I.

A GENERAL VIEW OF ASTRONOMY.

1. **A**STRONOMY in general comprehends the theory of the universe and the primary laws of nature, and is therefore a mixed science, composed of Physics and Mathematics.

2. The Physical part of Astronomy treats of those *laws* of matter and motion which regulate the various appearances of the heavenly bodies.

3. The mathematical part of Astronomy determines the *quantity* of either matter, space, or motion; such as ascertaining the magnitudes, distances, periods, eclipses, and other particulars of the heavenly bodies: and as these bodies are for the most part spherical, as well as the expanse in which they appear to move, it follows that the doctrine of spherics constitutes a principal part of the mathematics which is applied in astronomy.

4. It is easy to conceive, that in whatever part of the universe a spectator is placed, the heavenly bodies will appear to him as luminous points placed in a spheric

surface. Some will appear different to others, both in aspect and magnitude. This may arise either from a real difference of their magnitudes, or from their comparative distances from the place of observation. But as the eye must see the same distance in every direction where there is no obstruction, it follows that the extent of vision all round is terminated by a concave surface perfectly spherical, the centre of which is the place of the eye.

5. Astronomy is mostly distinguished into *Solar* or *Terrestrial*, according to the supposed situation of the spectator. These distinctions, however, are equally founded upon the Copernican system, as explained and confirmed by the discoveries of Kepler and Sir Isaac Newton, which system alone can account for the various appearances and phenomena of the universe.

6. By *Solar Astronomy* is understood the appearance of the heavens as seen from the centre of the sun. To an eye thus placed, the primary planets would be seen revolving round the concave expanse in orbits nearly circular and concentric, and in direct motion within the zodiac, but they would appear to move with different velocities; the satellites would be seen to revolve round their primary planets respectively; the comets would be seen to revolve round the sun in very eccentric orbits and in various and contrary directions, and the fixed stars only would always appear immoveable.

7. *Terrestrial Astronomy* explains the celestial appearances as seen from the earth. From this situation all the heavenly bodies seem to make an entire revolution from East to West in about a day; but this apparent
motion

motion of the concave expanse, or primum mobile, arises from the real motion of the earth round its axis, the sun and fixed stars only appear to move uniformly: the primary planets are sometimes seen to move in direct motion, that is, from West to East, according to the order of the signs; sometimes they are seen in retrograde motion, and sometimes stationary. The secondary planets appear of course to move analogous to the primary; but these particulars, with all the other doctrine and phenomena of the heavens, can only be detailed in a compleat system of astronomy.

8. The object here is to explain that part of Terrestrial Astronomy, which teaches the projection and computation of the whole or any part of that spheric surface, or primum mobile, which invests the earth, and in which the heavenly bodies appear to describe great and lesser circles in every diurnal revolution. This concave expanse is represented by the celestial globe, and being supposed concentric with the earth, it has correspondent circles drawn upon its surface.

9. As this part of Terrestrial Astronomy is performed by the doctrine of spherics, it is sometimes called Spheric Astronomy, or the Astronomy of the Sphere. It is also called Nautical Astronomy, as the most important problems of navigation are either performed by, or deduced from, spheric operations.

10. The one great end of all nautical calculations is to find *the place where the ship is*. This is no more than to find the latitude and longitude, which is performed either by celestial observations, or by the dead reckoning that is obtained from the log line and mariner's compass.

pafs. Now almost all celestial observations depend on fpheric principles; and even the variation of the compass is afcertained by azimuths and amplitudes, which are determined by fpheric folutions.

11. In order to know the true principles and foundation of fuch astronomical problems, it will be neceffary to project the fphere, and then to confider that part of it which is the fubject of enquiry. The projections of the fphere depend upon a proper application of the ftereographic problems, and alfo a knowledge of the following points, circles, arcs, and angles of the fphere; thefe are here defined preparatory to the projections, although it is prefumed that the learner is already acquainted with the outlines of aftronomy and the ufe of the globes.

SECTION II.

DEFINITIONS OF THE PRINCIPAL POINTS OF THE SPHERE.

1. THE *Centre* of the fphere is a point equidiftant from all parts of its furface.
2. *Zenith* is the vertical point of the fphere, that is, the point directly over head.
3. *Nadir* is that under point of the fphere diametrically oppofite to the zenith.
4. *Poles* are two immoveable points round which the fphere is fupposed to move, and a right line drawn from pole to pole is called the axis.

Poles

Poles of the Ecliptic are two point 90° from this great circle, and are each $23^\circ \frac{1}{2}$ from the poles of the equinoctial. All other circles also have their poles (xv).

5. *Cardinal Points* are four, namely, two equinoctial and two solstitial.

6. *Equinoctial Points* are the two opposite points of the sphere where the equinoctial and the ecliptic intersect: these are the first points of Aries and of Libra. The sun appears to enter the former about the 21st of March, and the latter about the 23d of September. The days and nights are then of an equal length in all latitudes, whence these points are called the equinoxes, one vernal and the other autumnal.

7. *Solstitial Points* are two points of the ecliptic where the sun's declination is greatest, both North and South. The northern or summer solstice is the first point of Cancer, upon which the sun appears to enter about the 22d of June; and the southern or winter solstice is the first point of Capricorn, where the sun enters about the 22d of December. At those seasons the days and nights are most unequal, and the nearer the poles the greater the inequality.

SECTION III.

DEFINITIONS OF GREAT CIRCLES.

1. *EQUATOR* is equidistant from both poles, dividing the earth into two equal parts.

2. *Equinoctial* is a circle of the celestial sphere equidistant from both poles, and corresponding with the equator of the earth.

3. *Meridians*, or *Hour Circles*, are circles perpendicular to the equator, and passing through the poles. These are also called *Circles of Right Ascension*, and *Circles of Terrestrial Longitude*.

4. *Ecliptic* is the circle through which the sun seems to move in a year. It cuts the equinoctial at an angle of $23^{\circ} 28'$ nearly. It is divided into 12 equal parts, called signs; these are thus marked and named:

Aries	Taurus	Gemini	Cancer	Leo	Virgo
♈	♉	♊	♋	♌	♍
Libra	Scorpio	Sagittarius	Capricornus	Aquarius	Pisces
♎	♏	♐	♑	♒	♓

5. *Solstitial Colure* is that meridian which passes thro' the solstitial points.

6. *Equinoctial Colure* is that meridian which passes through the equinoctial points.

7. *Circles of Celestial Longitude* are great circles perpendicular to the ecliptic passing through its poles.

8. *Horizon* is that great circle equally distant from the zenith and nadir, and which divides the upper from the lower hemisphere. This is called the rational Horizon, to distinguish it from the sensible Horizon, a lesser circle, which terminates the view of a spectator in certain situations upon the earth.

9. *Azimuth*, or *Vertical Circles*, are circles which pass through the zenith or nadir, and cut the horizon at right angles.

10. *Prime Vertical* is that azimuth circle which cuts the horizon in the East and West points.

11. The *Azimuth Circle*, which is perpendicular to the Prime Vertical, cuts the horizon in the North and South points, and is called the meridian of the place.

12. The *Meridian of the place* is also called the Twelve o'Clock Hour Circle; and the great circle which is at right angles with the meridian is called the Six o'Clock Hour Circle.

SECTION IV.

DEFINITIONS OF LESSER CIRCLES.

1. *PARALLELS of Latitude* are circles parallel to the equator on the earth.

2. *Parallels of Declination* are parallel to the equinoctial in the heavens.

3. *Parallels of Celestial Latitude* are parallel to the ecliptic.

4. *Parallels of Altitude*, or *Almacanthars*, are circles parallel to the horizon. Such is the sensible horizon, which bounds the view, and which is one of the most useful circles in astronomy; for to this circle, which is the only apparent one, most of the celestial motions are referred; such as the rising and setting of the sun and stars, their apparent altitudes, with many other useful particulars.

5. *Crepusculum*

5. *Crepusculum* is an almucanthar 18 degrees below the horizon, where the twilight begins and ends.

6. *Tropics* are parallels of latitude $23^{\circ} 28'$ on each side of the equator: the tropic of Cancer toward the North, and the tropic of Capricorn toward the South.

7. *Polar Circles* are $23^{\circ} 28'$ from the poles of the world; the Arctic circle toward the North, and the Antarctic toward the South.

SECTION V.

DEFINITIONS OF ANGLES AND ARCS OF CIRCLES.

1. *ALTITUDE* (of the sun, stars, &c.) is an arc of the azimuth circle between the object and the horizon.

2. *Zenith Distance* is the complement of the altitude of an object. Or it is an arc of the azimuth circle between the object and the zenith.

3. *Meridian Altitude*, and *Meridian Zenith Distance*, is the altitude or zenith distance when the object is on the meridian of the place.

4. *Azimuth* is an angle at the zenith formed by the vertical or azimuth circle of any given object and the meridian of the place. The azimuth is measured by the arc of the horizon which is intercepted between the vertical circle of the object and the meridian of the place.

5. *Amplitude*

5. *Amplitude* is an arc taken between the East or West points of the horizon and the centre of a celestial object at its rising or setting. Or the Amplitude may be defined, an angle at the zenith contained between the prime vertical and an azimuth circle passing through the object at its rising or setting.

6. *Right Ascension* is an arc of the equinoctial intercepted between the sun or star's meridian, or place in the heavens, and the first point of Aries.

7. *Oblique Ascension* is an arc of the equinoctial intercepted between the first point of Aries and the eastern point of the horizon when the object is rising.

8. *Oblique Descension* is an arc of the equinoctial intercepted between the first point, or Aries, and the western point of the horizon when the object is setting.

9. *Ascensional Difference* is an arc of the equinoctial intercepted between the sun or star's meridian and that point of the equinoctial that rises with the object.

10. *The Sum or Difference of the Right Ascension, and the Ascensional Difference*, is the oblique ascension or descension.

11. *Declination* is an arc of the meridian between the equinoctial and the object. If the object be on the North side of the equinoctial it is called North Declination, if on the South it is called South Declination.

12. *Latitude* of any point, or object in the heavens, is an arc of a circle of celestial longitude intercepted between the object and the ecliptic, and it is called
North

North or South Latitude as the object is North or South of the ecliptic.

13. *Latitude* of a place on the earth is an arc of the meridian between the equator and the place, and corresponds with declination in the heavens.

14. *Longitude* of any point or object in the heavens is an arc of the ecliptic intercepted between the first point of Aries and a circle of longitude passing through that point.

15. *Longitude* of a place on the earth is an arc of the equator between a given meridian and the meridian of the place. This arc of the equator measures its corresponding angle of longitude at the pole.

16. *Hour of the Day* is an arc in the heavens resembling an arc of longitude upon the earth. It is an arc of the equinoctial intercepted between the meridian of the place and the meridian which passes through the sun's place. This arc measures the angle which those two meridians make at the pole, and which is called the *Horary Angle*. The horary angle at 12 o'clock is nothing, because the meridian of the place and the sun's meridian coincide. At 1 or 11 o'clock the horary angle is 15 degrees. And thus the angle may be found by the time, or the time by the angle. As 24 hours is to 360 degrees, so is any number of hours to the horary angle: or as 360 degrees is to 24 hours, so is any number of degrees to the hour of the day, in the same manner are longitude and time reciprocally converted into each other.

17. *Elevation of the Pole* is an arc of the meridian contained between the pole and the horizon. This arc
is

is always equal to the latitude of the place, or the distance of the zenith from the equinoctial.

18. *Elongation* is the angular distance of a planet from the sun as seen from the earth.

19. *Lunar Distance* is the arc of a great circle, which shews the nearest distance between the centre of the moon and the centre of the sun or a star.

20. *Obliquity of the Ecliptic* is the angle made by the intersection of the equator and ecliptic, and is measured by the intercepted arc of the solstitial colure; which arc is the sun's greatest declination, and is now $23^{\circ} 28'$ nearly. The obliquity of the ecliptic is found to decrease about $\frac{1}{2}$ a second annually.

21. *Diurnal Arcs* are those parts of the parallels of declination of any celestial object described between the times of its rising and setting. And *Nocturnal Arcs* are the remaining arcs of those lesser circles which are described from the time of setting to the time of rising.

22. *Semi-diurnal and Semi-nocturnal Arcs* are the halves of the Diurnal and Nocturnal Arcs. The Semi-diurnal Arc is measured by the corresponding part of the equator, which shews the time between noon and the rising or setting of the sun. In the same manner the semi-nocturnal arc is measured by its corresponding arc of the equator. The degrees of any arc on the equator are turned into time by the hour circle, or may be converted into time by the 16th of this Section.

SECTION VI.

A SHORT EXPLANATION OF CERTAIN ASTRONOMICAL TERMS, WHICH DO NOT COME UNDER THE DESCRIPTION OF POINTS, CIRCLES, ARCS, OR ANGLES OF THE SPHERE.

ABERRATION, an apparent motion or change of place in the heavenly bodies, occasioned by the progressive motion of light combined with the earth's annual motion in its orbit.

Anomaly, an irregularity in the motion of a planet in its orbit. It is distinguished into *mean*, *eccentric*, and *true*.

Mean, or Simple Anomaly, is that distance which would take place if the planet moved uniformly in the circumference of a circle.

Eccentric Anomaly is an arc formed between the Apogee, or Aphelion, and a line drawn from the centre of the planet's orbit through the planet, and continued to the length of half the line of apfides.

True or equated Anomaly is the true distance of a planet from its aphelion. Or it is the angle formed at the sun by a line drawn from the sun to the planet, and by the line of apfides; the former line is called the Radius Vector.

Antecedentia, the apparent motion of a planet from East to West contrary to the order of the signs, as from Taurus towards Aries.

Aphelion, or *Aphelium*, that point in the orbit of the earth or a planet in which it is at the greatest distance from the sun. The opposite point of the orbit, in which it is nearest to the sun, is called the *Perihelion*.

Apogee, or *Apogeeon*, that point in the orbit of the sun, moon, &c. in which it is at its greatest distance from the earth. The opposite point of the orbit, in which it is at its nearest distance, is called the *Perigee*.

The Apogee of the sun is the same as the Aphelion of the earth, and the Perigee of the sun the same as the Perihelion of the earth.

Apses, or *Apsides*, are the two points in the orbit of a planet where it is at its greatest or least distance from the sun or earth. The farthest is called the *higher Apse*, and the nearest the *lower Apse*. The former is mostly called the *Aphelion* or *Apogee*, and the latter the *Perihelion* or *Perigee*.

A right-line or diameter which joins these two points is called the *Line of Apsides*. It is the *longest* or *transverse diameter* of the ellipsis or orbit of the planet.

Armillary Sphere, an artificial sphere composed of a number of circles, which represent the several great and lesser circles, &c. of the universe, such as are defined in the foregoing Sections.

Attraction, or *Gravitation*, is that universal property of matter by which all bodies mutually tend to each other.

The principle or law by which two bodies tend to unite is always in a ratio *directly as their mass*, and *inversely as the squares of their distances asunder*. This law of gravitation,

gravitation, and that by which a body put into motion, and meeting with no resistance, will for ever move in the same direction, form the theory of the motion of the heavenly bodies.

Centre of Attraction is the point to which bodies tend by their gravity, or that point to which a revolving planet or comet is impelled or attracted by the force or power of gravitation.

Centrifugal Force is that force by which a revolving body endeavours to fly off or recede from its centre of attraction. This is supposed the original impetus given to those bodies by which each would fly off in a direct line or tangent if not counteracted by the power of attraction.

Centripetal Force is that force by which a moving body is drawn toward the centre of attraction.

The compound and counteracting effects of centrifugal and centripetal forces constitute the principles which regulate the motions of the planets. If these forces were equal, the orbits of the heavenly bodies would be circular; but it is found that their powers preponderate reciprocally, and therefore the orbits of the planets are elliptical and of various eccentricities. The following two general laws regulate all their motions, namely,

1st. *The planets describe equal areas in equal times.*

2d. *The squares of the times of the revolutions of the planets are in proportion to the cubes of their distances from the sun.*

Comets

Comets, certain bodies which move round the sun in very eccentric orbits and in various directions. They are distinguished from the planets by their eccentricity, as well as by their tails, which are of a hairy, nebulous appearance.

Conjunction is when two heavenly bodies, seen from the sun or the earth, appear in the same degree of the ecliptic.

Constellation is a number of stars lying contiguous, which astronomers, to aid the memory, suppose to be circumscribed by the outlines of some animal or other imaginary figure.

Cosmical rising or setting of the stars is when they rise or set with the sun.

Consequentia, a direct motion of the planets according to the order of the signs, as from Aries towards Taurus.

Culminating, a term applied to a celestial object when it comes to the meridian of a place.

Cycle, some certain number of revolutions of a heavenly body.

Cycle of the Moon, a period of nineteen years, at the end of which time the lunar aspects, conjunctions, &c. commence, and continue nearly the same as they were the nineteen years before. The lunar cycle is also called the Metonic cycle, from Meton, the name of the inventor.

Cycle of the Sun, a period of 28 years, in which all the varieties of the dominical letters happen, and then return in the same order as they did 28 years before.

Day (natural) the time in which the earth compleats one entire revolution upon its axis.

Day (artificial) the time between the sun's rising and setting.

Day (astronomical, or solar) the time between two successive transits of the sun's centre over the same meridian, which always takes place at noon.

Day (sideral) is that portion of time between two successive transits of a star over the same meridian. The sideral day is $3^{\circ} 56'' 55$ less than a solar day: for in this time the sun appears to move $59^{\circ} 8'' 3$ in the ecliptic, which is its daily progress; whereas the star has no motion in the ecliptic either real or apparent.

Disc, the face of the sun or moon, which appears flat on account of its immense distance.

Digit, the twelfth part of the sun or moon's diameter, a term generally used in computing eclipses.

Dominical Letter is one of the first seven letters of the alphabet, used in almanacs, &c. to distinguish the Sundays.

Eccentricity, the distance between the focus of an ellipsis and its centre. The orbits of the planets and comets are elliptical, and the sun is always in one of the foci; hence the greater the disproportion between the transverse and conjugate diameters of the ellipsis, the greater the eccentricity.

Eclipse of the Sun, an obstruction of his light, occasioned by the interposition of the moon between the sun and earth.

Eclipse

Eclipse of the Moon, an obstruction of her light, occasioned by the interposition of the earth between the sun and moon.

An eclipse of the sun can only happen about the time of new moon, and an eclipse of the moon about the time of full moon.

The reason that there is not an eclipse at every opposition and conjunction of the sun and moon, is the obliquity or inclination of the plane of the moon's orbit to that of the ecliptic, which is about $5^{\circ} 18'$. The two points of intersection of the orbits of the sun and moon are called the nodes; and an eclipse can only happen at a conjunction or opposition, when the moon is in or near her nodes.

Ellipsis, or *Oval*, a figure formed by cutting a cone obliquely to its axis, so as not to touch the vertex or the base. The orbits of the primary and secondary planets, and of the comets, are ellipses more or less eccentric.

Epafl, the moon's age at the end of the year, or the difference between a lunar and solar year.

Equations, certain quantities by which the inequalities of the motions of a planet are estimated.

Equations of the moon are more numerous than those of any other body, being subject to more irregularities.

Equation of Time is the difference between apparent time and equal time, or the difference between a perfect time-keeper and a sun-dial. This difference between mean and true time is produced by the combined effect

of the obliquity of the ecliptic and the eccentricity of the earth's orbit.

Focus, one of the centres of an ellipsis.

Galaxy, or the *Milky Way*, a large irregular zone or luminous band which encompasses the heavens. This is attributed by *Herschel* and others to the closeness of the stars; but *La Lande* seems to think that there is beside these nebulae a luminous matter, which contributes to illuminate or whiten these parts.

Geocentric Place of a planet is its place in the ecliptic as seen from the earth.

Golden Number is the number of years elapsed in the lunar cycle, proceeding from 1 to 19, and is used to find the age of the moon.

Heliacal Rising of a star is when it is seen before sunrise, just beginning to emerge out of the sun's rays.

Heliacal Setting of a star is when it is so immersed in the sun's rays as to be invisible at sun-set.

Heliocentric place of a planet is its place in the ecliptic as seen from the sun.

Inferior Planets are those which move within the orbit of the earth, as Mercury and Venus. The Superior planets are those without the orbit of the earth, as Mars, Jupiter, Saturn, and Georgian.

Inferior Conjunction of a planet is when it is between the earth and the sun. Superior Conjunction, when the sun is between the earth and the planet. Venus, Jupiter, &c. are evening stars about the time of inferior conjunction,

conjunction, and morning stars about the time of superior conjunction.

Magnitudes of the fixed stars, their different apparent sizes, which is probably caused by their distances. They are classed into seven magnitudes; the largest are called the stars of the first magnitude, the next size are called stars of the second, and so on. The magnitudes are distinguished in the artificial globe by the seven first letters of the Greek alphabet.

Medium Cæli, or Mid Heaven of any place, is the degree of the ecliptic over the meridian of that place. Or the Mid Heaven is the distance of the meridian from the first point of Aries.

Month, a measure of time, and is either periodical, synodical, or solar.

A *Periodical* (sometimes called a *Lunar*) *Month*, is the time the moon takes in revolving from one point of her orbit to the same point again, and consists of 27 d. 7 h. 43 m.

Synodical Month is the time between one conjunction of the sun and moon and another, which is 29 d. 12 h. 44 m. This difference of 2 d. 5 h. 1 m. between the synodical and periodical month is occasioned by the earth's annual motion in the ecliptic. For while the moon is passing from her former conjunction with the sun round again to the same point of her orbit, the earth has proceeded forward in its annual course, leaving the moon as it were behind it; so that in order to complete her next conjunction with the sun, the moon must proceed nearly 30 degrees further in her orbit.

A *Solar* (or *Calendar*) *Month*, is the time which the earth takes to move through one of the signs of the zodiac, which at a mean is about $30\frac{1}{2}$ degrees.

Nebulæ, clusters of small stars which have been chiefly discovered by the telescope: they are so called from their cloudy appearance.

Nodes, two opposite points where the orbit of a planet intersects the plane of the ecliptic.

Nutation, or *Deviation*, a certain tremulous motion of the earth's axis, which makes the fixed stars appear to change their positions about 9"; its period is eighteen years. This effect is produced by the attraction of the moon on the earth, nearly in the same manner that the precession of the equinoxes is caused by the joint attraction of both the sun and moon.

Nonagesimal Degree, the highest point of the ecliptic at any given time, and is 90° from its intersection with the eastern or western points of the horizon.

Orbit, the path of a planet in its annual course.

Parallax is the difference of the place of any celestial object, as seen from the surface or from the centre of the earth.

Parallax in Altitude means the effect in any given altitude. See *Refraction*.

Horizontal Parallax is the effect of parallax in the horizon.

Parallax of the earth's annual orbit is the angle at any planet which is subtended by the distance between the sun and the earth.

Penumbra,

Penumbra, a faint shadow which borders or furrounds the dark shade cast by an eclipse.

Perigeon, that point of a planet's orbit in which the planet is at its least distance from the earth.

Peribolion, that point of a planet's orbit in which the planet is at its least distance from the sun.

Phases, the different appearances of the moon or planets, according as more or less of their illuminated hemispheres are presented to the sight.

Planets, bodies revolving within the solar system.

Primary Planets, bodies that regard the sun as the centre of their motions.

Secondary Planets, those bodies which regard the primary planets as their centres of motion.

Precession of the Equinoxes, a motion of the two points where the equator intersects the ecliptic. These equinoctial points are found to go backward about 50 seconds every year, or one degree in 72 years, so that in the space of 2160 years these points change a whole sign in the zodiac, and in 25920 years return to the same point, performing what is called *the Grand Celestial Period*.

Primum Mobile, that great concave expanse or sphere which appears to turn round the earth as a centre every 24 hours, and to carry with it all the heavenly bodies.

Refraction is a bending which the rays of light suffer in passing through mediums of different densities.

Both Refraction and Parallax produce the greatest effect at the horizon, and these effects gradually diminish to the zenith, where they become nothing. Refraction makes bodies appear higher than they really are, and Parallax makes them appear lower.

Secular Motion is the quantity which a planet at the end of some ages is more or less advanced than it would be, if the annual revolutions had been always the same. These secular inequalities are chiefly perceivable in Saturn, Jupiter, and the Moon.

Stars, those bodies which shine by their own light, and remain fixed or immoveable.

The stars are not all in reality *fixed*: some change their positions and aspects at different periods: new ones appear and old ones disappear: others appear and disappear at stated times, and are called periodical stars: some are double, others are in clusters, and give a white appearance to that part of the heavens where they are seen. See *Galaxy*.

Beside the real changes in the positions of *some* stars, *all* have apparent changes of place from *Aberration*, *Nutation*, and *Parallax*.

Syzigies, those points of the moon's orbit in which she is at the time of conjunction and opposition, or at the time of new and full moon.

Vector Radius, or *Radius Vector*, a line supposed to be drawn from any planet to the sun, which moving with the planet describes equal areas in equal times.

Year,

Year, the principal measure of time, which is the space taken up by the sun in making its apparent revolution through the twelve signs of the zodiac.

The time which the sun takes in moving from any star in the ecliptic to the same star again is 365 d. 6 h. 9' 11", and is called a *Sideral Year*.

The time which the sun takes in moving from one equinoctial point in the ecliptic to the same point again is 365 d. 5 h. 48' 48", and this is the common, or *Solar year*.

The solar year being shorter than the sideral by 20' 23", shews that the sun comes to the same point again in the ecliptic before he has made a complete revolution with regard to the stars; consequently the equinoctial point, and indeed every point of the ecliptic, must have moved backward, or in *antecedentia*, and this motion is called the Precession of the Equinoxes. This 20' 23" of time turned into minutes and seconds of the great circle of the ecliptic will be 50' 3" nearly, which is the annual precession of the equinoctial point. Thus, as 365 d. 6 h. 9' 11" : 360° :: 20' 23" : 50' 3".

The *Lunar Year*, used by some of the ancients, was 12 moons, about 354 days.

Zodiac, a zone or girdle of about 18° broad, which furrounds the heavens, and in the middle of which is the ecliptic. The orbits of the planets are contained within the zodiac.

Zone is a division of the terrestrial globe contained between two parallels of latitude.

There

There are five Zones; two frigid, two temperate, and one torrid.

The Frigid Zones extend each $23^{\circ} \frac{1}{2}$ from the poles.

The Torrid Zone extends $23^{\circ} \frac{1}{2}$ nearly on each side of the equator.

The Temperate Zones are those spaces between the frigid and the torrid.

The learner will find it greatly to his advantage to get the foregoing Definitions all by heart, as well as the following Table: this will not only facilitate the acquisition of the subsequent part of this work, but will be highly useful as preparatory to the study of a regular system of astronomy.

A TABLE of the PERIODS, &c. of the Planets.

	Annual Period round the Sun.		Diurnal Rotation on its Axis.	No. of Moons.
	Solar Year.	Sideral Year.		
Sun			25 d. 10 h..	
Mercury	d. h. min. sec. 87 23 14 32.7	d. h. min. sec. 87 23 15 43 6	Unknown.	0
Venus	224 16 41 27.6	224 16 49 10.6	{ by some 24 d. 8 h. by others 23 h. 20'	0
Earth	365 5 48 48	365 6 9 11.6	24 hours.	1
Mars	686 22 18 27.4	686 23 30 35.6	24 h. 39' 21 $\frac{2}{3}$	0
Jupiter	4330 14 39 2	4332 14 27 10.8	9 h. 55' 40"	4
Saturn	10746 19 15 16.5	10759 1 51 11.2	{ by some 6 h. by others 10 h,	7
Georgian	83 yrs. 52 d. 4 h.	83 yrs. 150 d. 18 h.	Unknown.	2

A TABLE of the MAGNITUDES, DISTANCES, &c. of the Planets.

	Marks.	Dist. from the Sun in Eng. Miles.	Diameter in Eng. Miles.	Comparative Size with respect to the Earth.	
Sun	☉		887693	1384462	nearly 1400000 times as large.
Mercury	☿	36668373	3241	0,06456	the 15th part
Venus	♀	68518044	7639	0,89025	less by one ninth
Earth	☿	94725840	7962	1	
Moon	☾	fr. ☿ 239980	2174	0,02036	the 49th part
Mars	♂	144588575	4142	0,14060	the seventh part
Jupiter	♃	492665207	86488	1281,	1300 times as large
Saturn	♄	903690197	79491	995,	1000 times as large
Georgian	♅	1822568000	34500	80,49	80 times as large

A TABLE of the CONSTELLATIONS.

	Number of Constellations	Number of Stars.	Magnitudes.					
			I	II	III	IV	V	VI & VII
In the Zodiac	12	894	6	12	38	100	169	569
Northern Constellations	36	1243	5	21	92	200	291	639
Southern Constellations	32	706	9	32	75	185	188	217
Number of Stars in the 80 Constellations	-	2843	20	65	205	485	648	1420

The above 2843 stars are those only which are visible to the naked eye, but the number of telescopic stars encreases with the magnifying power of the instrument.

Dr. Herschel, with a telescope which magnifies about 5000 times, reckoned 44000 stars in the space of six degrees in length and three in breadth; this would make 75 millions in the whole heavens, if all the other parts contained stars in the same proportion.

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cations

SECTION VII.

PROJECTIONS OF THE SPHERE.

1. IN order to obtain the most clear and comprehensive idea of the various projections of the sphere, a celestial globe should be set in a position to correspond with each projection, (see page 28 and 42.) This method will be found well calculated to simplify the study of spheric astronomy, and notwithstanding the great and obvious utility of such an illustration, no writer on the subject has hitherto availed himself of it in any regular or systematic way.
2. The positions of a globe are generally distinguished into *Right*, *Oblique*, and *Parallel*.
3. A *Right Sphere* is when a meridian circle coincides with the horizon, and the zenith and the nadir are in opposite points of the equator. See Fig. 35.
4. An *Oblique Sphere* is when one pole is elevated above the horizon, and the other depressed below it. See Fig. 36 or 38.
5. A *Parallel Sphere* is when the equator coincides with the horizon, and the poles are the zenith and nadir. See Fig. 37.
6. In projecting the sphere in any of those positions, the primitive circle may represent that great circle which coincides with the horizon. Thus a right sphere is said to be projected *on the plane of the meridian*, an oblique

oblique sphere *upon the plane of the horizon*, (of the given latitude) and a parallel sphere *upon the plane of the equator*. These circles coincide with the wooden horizon on a globe set in any of the said positions.

The projection of the sphere upon the plane of a meridian circle is generally found the most simple and convenient. It is not necessary, however, that the primitive circle should always represent the meridian which coincides with the wooden horizon on the globe; for in an *oblique sphere* this projection is used with equal advantage. Thus by elevating the pole and viewing the globe sideways, the meridian circle which coincides with the brazen meridian may be represented by the primitive circle in the projection, and the eye is here supposed in that part of the equator which crosses the horizon. In Fig. 35, if the primitive or meridian circle SEPQ be supposed to coincide with the horizon (as a right sphere) the centre γ is the zenith; but if the same circle be supposed to coincide with the brazen meridian, and the pole P be elevated to the latitude RP, then Z will be the zenith and N the nadir.

PROJECTION 1.—FIG. 35.

To project the Sphere upon the Plane of a Meridian, suppose that which coincides with the Solstitial Colure.

Here the solstitial colure is first drawn as the primitive circle, and the equinoctial point is the place of the eye.

1. With the sweep of 60 from the line of chords describe the primitive circle PESQ. This represents the solstitial colure, the centre or pole of which is γ .

2. Draw

2. Draw the diameter or right circle EQ as an equator, and at right angles to it draw the equinoctial colure PS. This sign may also represent the axis of the globe, the extremities of which are P and S, the north and south poles.

3. *To draw the Meridians or Hour Circles.*

Let the diameter EQ be produced as a line of measures. Lay off from the centre both ways the tangents 15° , 30° , 45° , 60° , 75° , respectively, and they will give the centres of circles to be described through P and S, or the secants 15° , 30° , &c. will give the radius of each circle (Prob. 6, Page 35.)

These circles will cut the equator in points representing the 24 hours, the primitive or solstitial colure being the 12 o'clock hour circle, and PS the equinoctial colure the 6 o'clock hour circle.

4. *To draw the Parallels of Latitude or of Declination.*

Let PS be produced as a line of measures. Then suppose the required parallels of latitude or declination should be 10° asunder.

From the centre γ lay off the semi-tangents 10° , 20° , 30° , &c. both ways, on PS. Or,

From Q and P lay off the chords 10° , 20° , 30° , &c. both ways on the primitive.

Then the tangent of the complement 10° , 20° , 30° , &c. will be the radius of each lesser circle respectively.

Or, The secant of the complement of 10° , 20° , 30° , &c. respectively, laid off from the centre both ways

on PS, the line of measures, will give the centres of each corresponding parallel of declination (CXIV.) Thus are the lesser circles 10, 10, 20, 20, &c. drawn.

By the same rule the *tropics* ∞ a and ∞ b are drawn $23^{\circ} \frac{1}{2}$ on each side the equator. Also

The polar circles cc and dd at $66^{\circ} 30'$ on each side the equator.

Here the angle which the equator makes with the ecliptic is taken as $23^{\circ} \frac{1}{2}$, which is two minutes too much; but this difference can scarcely be perceived in the projection.

5. *To draw the Ecliptic.*

Lay off the chord $23^{\circ} \frac{1}{2}$ from Q to ∞ , and from P to ∞ on the primitive.

Through the centre ∞ draw the right circle ∞ ∞ , which will represent the ecliptic. Its poles are c and d, which are the intersections of the polar circles with the solstitial colure.

6. *To lay off the Signs upon the Ecliptic.*

The semi-tangent 30° and 60° laid off each way from the centre ∞ towards ∞ and ∞ respectively, will give the places of the signs.

7. *To draw Circles of Celestial Longitude, as c ∞ d.*

Let ∞ ∞ be continued as a line of measures. Then through c and d, the poles of the ecliptic, describe circles such as c ∞ d in the same manner as the hour circles were drawn through P and S.

The

The circle $c \approx d$ is described with the secant 60° , and therefore passes through the signs $\approx$ and $\dagger$. By this method also the signs of the ecliptic might be found.

8. *To draw Parallels of Celestial Latitude.*

These are lesser circles drawn parallel to the ecliptic, in the same manner that parallels of terrestrial latitude are drawn parallel to the equator.

9. *To draw the Horizon for Latitude $51^\circ 30'$.*

Lay off the chord $51^\circ 30'$ from E to Z and from Q to N. Draw ZN and HR at right angles to it.

Then HR represents the horizon, Z the zenith, and N the nadir, and the circle ZHN the meridian of the place.

10. *To draw Azimuth Circles, as ZAN.*

Azimuth or vertical circles are described like hour circles, to make any given angles with the meridian. Thus ZAN is described to make an angle of 50° with the meridian.

The great circle ZN being at right angles with the meridian is the azimuth circle, called the *Prime Vertical*.

12. *To draw Almicanthars or Parallels of Altitude.*

Almicanthars are drawn parallel to the horizon HR, in the same manner as circles of declination are drawn parallel to the equator; but these and parallels of celestial latitude are here omitted, as tending too much to crowd the figure.

PRO,

PROJECTION 2.—FIG. 36.

To project the Sphere upon the Plane of the Horizon of Latitude $51^{\circ} 30'$ North.

If a globe be set to the above latitude, and turned till the equinoctial points γ and $\sphericalangle$ coincide with the wooden horizon, and the northern portion of the ecliptic be uppermost; the problem is to describe the circles of the upper hemisphere of the globe in this position.

1. *To draw the horizon.*

With the sweep of 60 from the line of chords describe the primitive circle $xii\ E\ xii\ Q$, which represents the horizon, where the upper xii is the North, the lower xii the South, E the West, and Q the East points, Z the place of the eye and also the zenith.

2. *To draw Azimuth Circles.*

Great circles passing through Z the zenith, and cutting the horizon at right angles, are azimuth circles. Thus $xii\ xii$ is an azimuth circle and the meridian of the place, or 12 o'clock hour circle; EQ is another azimuth circle, which being at right angles with the meridian is the prime vertical.

3. *To find the North Pole P.*

The semi-tangent of $38^{\circ} 30'$, the complement of the latitude $51^{\circ} 30'$, laid off from Z toward the upper xii , will give P the pole.

4. *To draw the Six o'Clock Hour Circles EPQ .*

As this circle makes an angle with the horizon or primitive equal to the latitude of the place, let the secant

K

cant

cant $51^{\circ} 30'$ be laid off from P to G on the line of measures; then is G the centre of the fix o'clock hour circle EPQ, or the tan. of the lat. will extend from Z to G. Hence PG is the radius of the arc EPQ.

5. To draw the other Hour Circles.

Here EPQ must be conceived an arc of a primitive circle, PG its radius, and G a h ed a line of measures upon which the centres of the hour circles must be found.

And as the hour circles are 15° degrees asunder, the hour circle VP at the West side must cut the supposed primitive EPQ in P, so as to make an angle of 15° . The next circle iv P must make an angle of 30° with EPQ, and so on.

But as the radius GP of this primitive is greater than the line of chords on the Gunter, the centres a, b, &c. of the hour circles must be found by either of the two following methods.

First method of drawing the Hour Circles (by the Sector.)

Open the sector till the points 45° and 45° of the tangents be the distance of GP asunder (LXXXIX).

Then the transverse of the tangents 15° and 15° laid off from G, will extend to a on the line of measures, and a will be the centre of the hour circle VPV.

In the same manner the transverse $30^{\circ} 30^{\circ}$ will extend from G to b; that of $45^{\circ} 45^{\circ}$ from G to c, the centres of iv P iv and III P III. Then

To

To find the centres of 60° and 75° —The sector must be opened until the radius PG will extend transversely from 45° to 45° on the upper tangents (xc). Then the transverse 60° of these lines will extend from G to d on the line of measures, and d will be the centre of the hour circle $II P II$.

In the same manner the transverse 75° on the upper tangents will find the centre of the hour circle $I P I$, which centre goes beyond the limits of the paper.

The centres a, b, c , &c. being found on one side of G , they may be measured at the same distances on the other side, or they may be laid off on both sides of G at the same time.

Another method of drawing the Hour Circles (by Gunter's Scale).

With the sweep of 60 degrees from the scale of chords round P as a centre, draw the quadrant on the east side of the figure from below F at α to 90 degrees near V .

From the same line of chords lay off $15^\circ, 30^\circ, 45^\circ$, &c. up to 90 degrees.

Then a ruler on P and each of these figures on the quadrant will give the centres a, b, c , &c. in the line of measures.

This method is founded on Art. LXXIX, and is perhaps more practical than that by the sector; but both together are recommended by way of verification.

Drawing circles of this description generally embarrasses learners; the different methods are therefore the more particularly explained in these projections.

Note. If the lower hour circles were continued round, they would form a sphere (like Fig. 35) the radius of which would be GP, and the North pole P. This may be done where there is sufficient room.

6. *To draw the Equator EAQ.*

As the distance of the equinoctial from the zenith is equal to the latitude of the place, the semi-tangent $51^{\circ} 30'$ will extend from Z to A; the tang. of the complement of the latitude $38^{\circ} 30'$ laid off the other way from Z will give the centre of EAQ, or the secant of $38^{\circ} 30'$ is the radius of this circle and P its pole.

7. *To draw the Ecliptic E ∞ Q.*

As the ecliptic makes an angle of nearly $23^{\circ} \frac{1}{2}$ with the equinoctial, let this be subtracted from the latitude, and 28 degrees remain, the semi-tangent of which being laid from Z will extend to ∞ , and the tangent of its comp. 62° will extend from Z toward P in the line of measures, which shews the centre of the ecliptic, or the secant 62 degrees is the radius of the ecliptic.

8. *To draw the Polar Circles and Tropics.*

These operations are founded on Article xcv, and may be performed either by the chords on the primitive, or semi-tangents on the line of measures; the latter is the most practical, and is done by laying off from Z (the centre) the greatest and least distances of the required circle, then the middle between this interval is the centre sought.

Thus, to draw the polar circle p q, which extends $23^{\circ} \frac{1}{2}$ degrees round the pole P.

The

The pole P is $38\frac{1}{2}$ degrees (the complement of the latitude) from Z; from this subtract $23\frac{1}{2}$ degrees, and the remainder is 15 degrees; add $23\frac{1}{2}$ degrees to $38\frac{1}{2}$ degrees, and the sum is 52 degrees.

The semi-tangent 15° from Z will extend to q, and the semi-tangent 52° from Z will extend to p. Bisect the line pq, which will give the centre of the polar circle, and p is the pole of the ecliptic; for $Z \varpi = 28^\circ$, $Z p = 52^\circ$, and their sum $= 90^\circ$.

In the same manner the tropic of Cancer $c \varpi c$, and the tropic of Capricorn $d \varpi d$ are described.

Thus, the tropic of Cancer is $23\frac{1}{2}$ degrees from the equator on the North side; this subtracted from $51\frac{1}{2}$ degrees, the latitude, leaves 28 degrees, the least distance, that is from Z to Cancer.

Then to find the greatest distance, that is from Z to the tropic of Cancer the other way. From Z to P is $38\frac{1}{2}$ degrees, and from p to the tropic is $66\frac{1}{2}$ degrees, their sum $= 105^\circ$. Lay this off from the semi-tangents from Z on Z p continued, and mark the place it extends to. Bisect the distance between this mark and ϖ , which will give the centre of the tropic of Cancer $c \varpi c$.

To draw the tropic of Capricorn $d \varpi d$. Its least distance from Z is 28° , this added to 47° (the breadth of the torrid zone) $= 75^\circ$, which laid off from Z will extend to ϖ .

The greatest distance from Z the other way is $(105 + 47 =) 152$ degrees; let this be laid off upon Z p continued, and the place it extends to marked. Bisect the

distance between this mark and ϖ , and it will give the centre of $d \varpi d$, the tropic of Capricorn.

This operation is explained the more at length, being generally found difficult to learners.

9. *To draw Circles of Celestial Longitude.*

These are described through the pole p of the ecliptic, from centres found in the line BAC , in the same manner that the hour circles are described through the pole p of the equator from centres found in G, a, b, c , &c. (4).

The line BAC is found by laying off the tangent 28° from Z to A , or the secant 28 degrees from p to A ; for $p \times II$ at the upper part $= 28$ degrees, that is, $P \times II$ ($51 \frac{1}{2}$ degrees) $- p P$ ($23 \frac{1}{2}$ degrees) $= 28$ degrees.

BCA being drawn perpendicular to $xII \ xII$, open the sector to AP , and lay off both ways from A the transverse of the tangents 30° and 60° , these will give the centres required; thus A is the centre $\gamma p \triangle$. The tangent 30° both ways will give the centres of γp and πp , and 60° both ways upon the upper tangents will give the centres πp and ϱp . Where these circles of longitude cut the ecliptic, mark the signs accordingly.

10. *To describe Circles of Celestial Latitude.*

Circles of celestial latitude, such as $\gamma q \gamma$, are described round p , the pole of the ecliptic, in the same manner that circles of declination or parallels of latitude are described round p , the pole of the equator.

Thus, to draw a circle of 40 degrees of celestial latitude. The required circle must be 40 degrees from the ecliptic $E \triangle Q$, and of course 50° from its pole p .

Now

Now the pole p is 62° from Z , therefore the least distance of the parallel circle from Z must be 12° , and the greater 112° , the sum and difference of 50 and 62 .

From Z lay off semi-tangent 12° to q , and 112° in ZP continued. Bisect the distance between this point and q , and it will give the centre of $\gamma q \gamma$, the required parallel of celestial latitude.

11. *To draw Almicanthars or Parallels of Altitude.*

This is performed by Art. cxiii. for almicanthars being concentric with the primitive, they must be described about the pole Z with the semi-tangents of their distances from it. Thus the lesser circle $a b$ is described about Z with the semi-tangent 80 degrees, it being 10 degrees above the horizon.

PROJECTION 3.—FIG. 37.

To project the Sphere upon the Plane of the Equator, that is, to project a parallel Sphere.

Here the eye is supposed to be in one of the poles, thence viewing the northern hemisphere.

1. The primitive circle represents the *equator*.
2. The *Hour Circles* are represented by diameters or right circles, making angles of 15 degrees with each other, which angles are measured on the primitive from the scale of chords.
3. *Parallels of Terrestrial Latitude*, or circles of declination, are described in the same manner as parallels of altitude, that is, they are concentric with the primitive

or equator, and are described round the pole P with the semi-tangents of their several distances from it, or with the semi-cotangents of their degrees of declination, of latitude or of altitude. Thus p q, the arctic circle, and a ∞ , the tropic of Cancer, are described with $23^{\circ} \frac{1}{2}$ and $66^{\circ} \frac{1}{2}$ respectively, and the other parallel circles are all drawn at 10 degrees asunder.

4. *To draw the Ecliptic.*

This must be drawn to make an angle of $23^{\circ} \frac{1}{2}$ with the equator, or primitive; therefore the tang. of $23^{\circ} \frac{1}{2}$ laid off from P toward a will give O, the centre of the ecliptic $\gamma \infty \Delta$, and the pole of the ecliptic is p in the polar circle.

5. *To draw Circles of Celestial Longitude.*

These are described through the pole p of the ecliptic in the same manner as the hour circles were drawn through p, the pole of the equator (Art. 4, Projec. 2). Thus the tangent $66^{\circ} \frac{1}{2}$ laid off from P on P ∞ continued, will give the centre of H p R, and the centres of the other circles γ p, π p, μ p and Ω p, are found in a line drawn through this point perpendicular to P xii at the tang. 30° and 60° both ways from the centre of H P R.

Thus the signs γ , π , Ω , and μ , are marked on the ecliptic.

6. *To draw circles of Celestial Latitude.*

These are drawn round p, and parallel to the ecliptic, in the same manner as the circles of declination were drawn in Projection 2d. that is, by setting off the
I greatest

greatest and least distances of the proposed circle from P, and bisecting the interval, which will give the centre A.

7. To draw the Colures.

The meridian $xii\ P\ xii$ here represents the solstitial colure, and $vi\ P\ vi$ the six o'clock hour circle or equinoctial colure.

8. To describe the Horizon of London, $51^{\circ}\ 30'$.

The horizon of any place being inclined to the equator in an angle equal to the co-latitude, the tang. $38^{\circ}\ 30'$ laid off from P toward ∞ will extend to Z the centre of HOR, the horizon required.

9. To describe Azimuth Circles for the same Horizon.

Here the prime vertical makes an angle with the equator equal to $51^{\circ}\ 30'$ the latitude; the tang. of this laid off from p will extend to y the centre of H Z R, the prime vertical.

Any other azimuth circle, as Z A, making a given angle with the meridian Z O, is drawn from a centre found in a line drawn through y (the centre of H Z R) at right angles with the meridian $xii.xii$. These centres are found in the same manner as those of the hour circles in the last Projection. Thus, with the sector opened to P y the radius of the prime vertical, lay off from y the transverse tangent of the required angle. In the present instance the azimuth circle Z A makes an angle of 40° with the meridian, and x is the centre.

PROJECTION 4.—FIG. 38.

To project the Sphere upon the Plane of the Ecliptic.

Here the eye is supposed in P, one of the poles of the ecliptic, thence viewing the northern hemisphere.

1. *To describe the Ecliptic.*

The ecliptic in this Projection is represented by the primitive circle, the centre of which is p its pole.

2. *To draw Circles of Celestial Longitude, and find the Places of the Signs of the Zodiac.*

Circles of celestial longitude are here represented by diameters or right circles.

Those drawn 30 degrees asunder mark the divisions of the signs in the zodiac.

3. *To describe circles of Celestial Latitude.*

These are concentric to the ecliptic, and are therefore described about p. Such is the lesser circle, the diameter of which is a b, representing a parallel of 10° of celestial latitude.

4. *To draw the equator.*

The equator must be drawn to make an angle of $23^{\circ} \frac{1}{2}$ with the primitive, and therefore the tangent of $23^{\circ} \frac{1}{2}$ laid off from p toward ∞ will find q the centre of the equator H X I I R, and the half tangent of $23^{\circ} \frac{1}{2}$ laid off the same way will give P the pole of the equator.

5. *To draw the Colures.*

Here the line ∞ P ∞ represents the solstitial colure.

The

The equinoctial colure HPR makes an angle with the primitive or ecliptic of $66\frac{1}{2}$ degrees, and therefore its tangent laid off from p on p ω continued will find the centre of the equinoctial colure.

6. *To describe the Hour Circles.*

The hour circles are here described in the same manner as in Projection the 2d. Thus, the equinoctial colure is the six o'clock hour circle, through the centre of which draw a line at right angles with P ω continued.

With the sector opened to the radius of the circle HDR lay off the transverses 15° , 30° , 45° , &c. both ways, which will give the centres of the hour circles respectively, and they will cut the equator in the hour points.

7. *To draw Circles of Declination or of Terrestrial Latitude.*

The *Arctic Circle* p q is described by laying off the half tangent 47 degrees from p to q, and bisecting the interval which gives the centre.

The *Tropic of Cancer* 12.12 is described by laying off the semi-tangents of its greatest and least distance from p, and bisecting the interval. Thus the semi-tang. 90° the greatest distance will extend from p to ω , and the semi-tangent $43'$ ($66\frac{1}{2} - 23\frac{1}{2}$) its least distance, will extend from p downward to 12.

8. *To draw the Horizon for the latitude of London.*

Here the place required is $38\frac{1}{2}$ degrees from the pole P, and of course 15 degrees from the pole p, the
semi-

semi-tangent of which will extend to Z, the latitude of London.

Lay off the tangent 15 degrees from p toward ∞ which will give the centre of H O R the horizon.

9. *To draw Azimuth Circles.*

The prime vertical H Z R is described from a centre found in p ∞ continued, by laying off the tangent 75° from p (the complement of 15°).

Any other azimuth circles are described through Z, making given angles with the meridian Z O, by finding their centres in a line drawn perpendicular to Z ∞ and through the centre of H Z R. In the same manner the hour circles are described in Projection the 2d.

10. *To describe Almicanthars for lat. $51^\circ 30'$.*

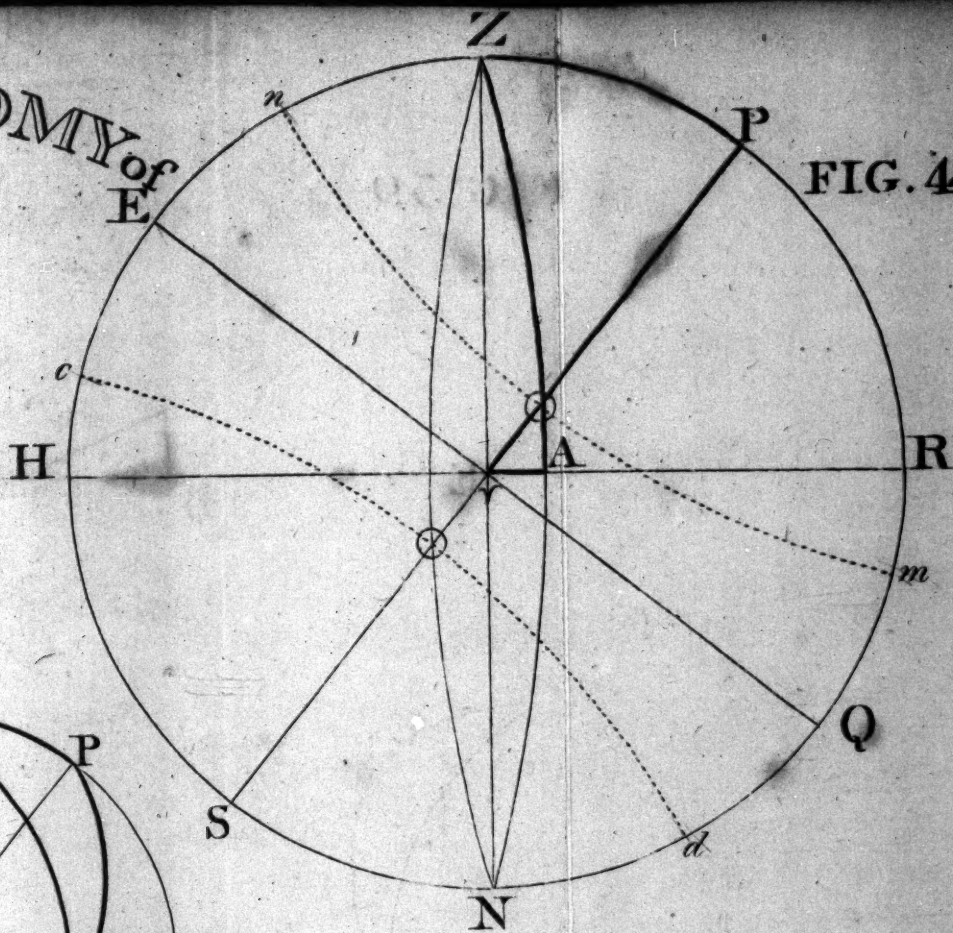
A parallel of altitude here is a lesser circle parallel to the horizon H O R; or what is the same thing, round the pole Z.

Thus, to draw an almicanthar c d c of 30 degrees altitude from the horizon, or 60 degrees from the pole Z. Lay off its greatest and least distance from p, that is, 45° and 75° , and find the centre of this interval, which is the centre of the almicanthar c d c.

SECTION

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FIG. 40.



G. 43.

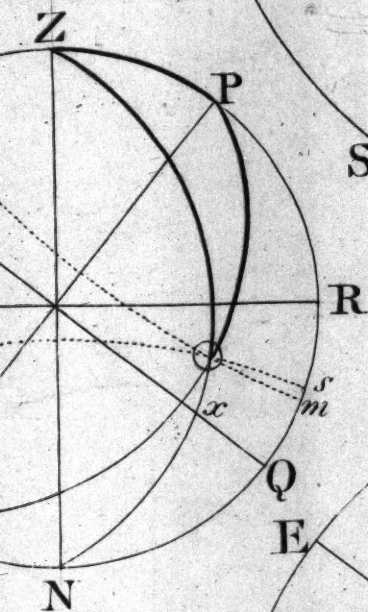
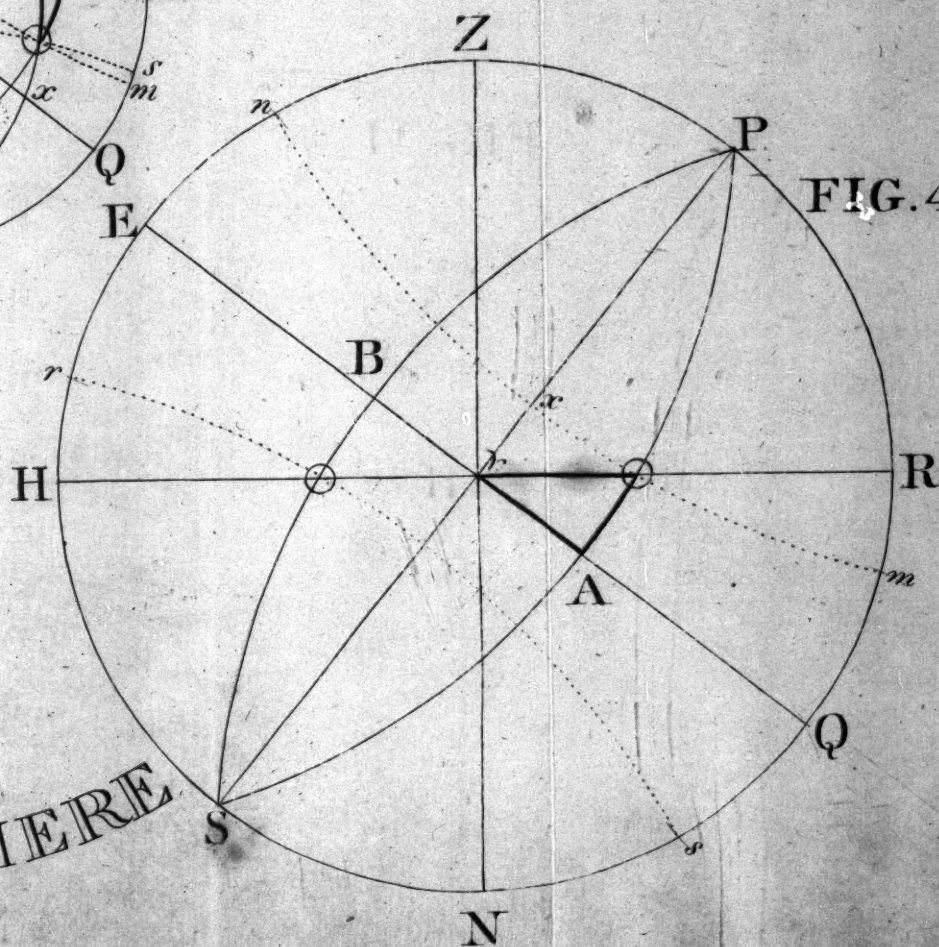


FIG. 42.



SPHERE

SECTION VIII.

ASTRONOMICAL PROBLEMS.

PROBLEMS in astronomy are most easily performed by first considering the subject upon a globe, and thence forming a rough solution. This will direct and illustrate the projection, and the projection will serve as a guide and a check to the calculation.

Operations of this kind are often rendered obscure by the confused idea which the learner has of his situation with respect to the universe, it is therefore here repeated that, *he should consider himself as at the outside of the concave expanse, where he seems to be placed in viewing the convex surface of a celestial globe.*

In assimilating the projection to the position of the globe when the pole is elevated, it is generally most convenient to make the great circle which coincides with the brazen meridian the primitive circle; but sometimes the projection may be made upon the plane of any great circle. That, however, should be always chosen which is performed with greatest simplicity of operation: and beside the application of what is laid down in the foregoing Section, the following rule should be observed: *Choose such a plane of projection, that a given side may be upon the primitive or a given angle at the centre.*

In the following projections small circles only are dotted, and those parts of the great circles which compose the given triangle are for the sake of distinction drawn stronger than the rest.

P R O-

PROBLEM 1. FIG. 39.

Given the sun's longitude and the obliquity of the ecliptic.

Required the sun's right ascension and declination.

Exam. On the 1st of May, 1795, the sun's longitude was 1 sign $11^{\circ} 1' 44''$, and the obliquity of the ecliptic $23^{\circ} 27' 51''$ *, required the rest.

1. By the Globe.

Find the sun's longitude, or place in the ecliptic, and bring it to the brazen meridian. Then the arc of the equator between the first point of Aries and the brazen meridian shews the sun's right ascension, and the arc of the brazen meridian between the equator and ecliptic, shews the declination.

2. By Projection.

This triangle will be most simple when constructed upon the plane of the solstitial colure, as the given angle (the obliquity of the ecliptic) will be then at the centre of the primitive, which is the first point of Aries.

Draw the primitive PESQ, with the diameters EQ for the equator and PS for the equinoctial colure.

Lay off $E\varpi = 23^{\circ} 27' 51''$ the obliquity of the ecliptic, and draw the diameter $\varpi\psi$ for the ecliptic.

* See the Nautical Almanac for May 1, 1795. The learner should as an exercise take out the sun's longitude and declination for any other day to find the rest, so as to make his calculations agree with those of the Nautical Almanac.

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From the scale S $\odot$ T lay off the sun's long. $41^{\circ} 1' 44''$ from γ to $\odot$ in the ecliptic, which is the sun's place.

Through the points P $\odot$ S draw a circle of right ascension (xcv) then will B $\odot$ γ be the triangle.

Here the angle at γ and the side $\gamma \odot$ are given, and the rest may be measured by the 4th and 5th of the Stereographic Problems.

3. By Calculation.

In the right-angled triangle $\gamma \odot$ B

Given the sun's long. $\gamma \odot = 41^{\circ} 1' 44''$.

Greatest declin. $\angle \odot \gamma B = 23^{\circ} 27' 51''$.

Required the right ascension γB and present declination B $\odot$.

To find the Declination (CXLII).

As rad.	90°	co. ar.
To s, sun's long.	$41^{\circ} 1' 44''$	9,8171947
So is s, sun's greatest decl.	$23^{\circ} 27' 51''$	9,6000745
To s, present declin.	$15^{\circ} 9' 7''$	9,4172692

To find the Right Ascension (CLXI).

As cot. sun's longitude	$41^{\circ} 1' 44''$	9,9396053	co. ar.
To rad.		10	
So is cos. obl. eclip.	$23^{\circ} 27' 51''$	9,9625158	
To tan. right ascension	$38^{\circ} 35' 51''$	9,9021211	

Here $38^{\circ} 35' 51''$ turned into time is 2 h. $34' 23'' 24'''$, the right ascension in time.*

PRO.

* The following rule for converting degrees into time, and the contrary, is more practical and concise than that given in page 108, though both are founded on the same principles.

1. To

PROBLEM 2. FIG. 39.

Given the obliquity of the ecliptic and the sun's present declination.

Required the sun's longitude and right ascension.

Exam. *The obliquity of the ecliptic being $23^{\circ} 27' 51''$, and the sun's present declination $15^{\circ} 9' 7''$, what is the sun's longitude and right ascension?*

1. By

1. To convert Degrees into Time.

Rule. Divide the given hours by 15, and multiply the remainder by 4 for minutes.

Divide the minutes and then the seconds in the same manner by 15, and multiply the remainders by 4, putting each quotient under its respective dividend.

2. To convert Time into Degrees.

Rule. Multiply the hours by 15, to which add one fourth of the minutes and of the seconds, each being removed one place higher.

EXAMPLE.

Reduce $38^{\circ} 35' 51''$ into time, and the contrary.

$$15 \overline{) 38 \ 35 \ 51}$$

$$2 \ 32$$

$$2 \ 20$$

$$3 \ 24$$

$$\hline 2 \text{ h. } 34' 23'' 24''$$

$$15$$

$$30$$

$$8 \ 30$$

$$5 \ 45$$

$$6$$

$$\hline 38^{\circ} 35' 51''$$

$$= 38^{\circ} \div 15$$

$$= 35' \div 15$$

$$= 51'' \div 15$$

The reason of these operations is that $15^{\circ} = 1$ hour, and therefore $18 = 4$ minutes.

$$= 34' \times 15 \div 60$$

$$= 23'' \times 15 \div 60$$

$$= 24'' \times 15 \div 60$$

Thus multiplying by 15 and dividing by 60, is the same as dividing by 4 and removing each a place higher.

1. By the Globe.

Mark the sun's present declination on the brazen meridian, and turn the globe till the ecliptic comes under the figure. Then will the distance from the meridian to the first point of Aries shew the sun's longitude on the ecliptic, and its right ascension on the equator.

2. By Projection.

The solstitial colure being described with the equator and ecliptic, and equinoctial colure, as in the last Problem, make En and Qn equal to the given declination, and round P describe the parallel of declination nn (cxiv). Where this intersects the ecliptic in $\odot$ is the sun's place. Through P $\odot$ S describe a circle of right ascension P $\odot$ S (xcv) then is $\Upsilon \odot B$ the triangle required, the unknown parts of which may be measured as before.

3. By Calculation.

In the right-angled spheric triangle $\Upsilon \odot B$,

Given the obliquity of the ecliptic $B \Upsilon \odot = 23^\circ 27' 51''$, and the declination $B \odot = 15^\circ 9' 7''$.

Required the sun's longitude, and right ascension.

To find the sun's longitude (cxlvi).

As s, obl. ecl. =	$23^\circ 27' 51''$	0,3999255
To s, declin. =	$15 \quad 9 \quad 7$	9,4172719
So is radius 90		10
To s, $\Upsilon \odot$ sun's long.	$41 \quad 1 \quad 44$	9,8171964

L

To

To find the sun's right ascension (CLXIII).

As radius	90°	co. ar.
To tan. declin.	15° 9' 7"	9,4326383
So is cot. obl. ecl.	23 27 51	10,3624413
To fine right ascen.	38 35 51	<u>9,7950796</u>

PROBLEM 3. FIG. 39.

Given the obliquity of the ecliptic and the sun's right ascension.

Required the sun's declination and longitude.

Exam. When the sun's greatest declination is 23° 27' 51", and his right ascension 38° 35' 51", what is his present declination and longitude?

1. By the Globe.

Bring the sun's right ascension in the equator to the brazen meridian, then will the intercepted arc of the ecliptic to Aries shew the sun's longitude, and that arc of the meridian which is between the sun's right ascension and longitude will shew the present declination.

2. By Projection.

The solstitial and equinoctial colures being described as before with the equator and ecliptic, make $\gamma B =$ the given right ascension, and describe the circle PBS (xcv) which will cut the ecliptic in $\odot$ the sun's place.

3. By Calculation.

In the right-angled spheric triangle $\gamma \odot B$ the angle γ is given = 23° 27' 51", and the leg $\gamma B = 38° 35' 51"$, to find $\gamma \odot$ and $B \odot$.

To

To find the declination $B \odot$ (CLXV).

As cot. $\angle \gamma$	$23^{\circ} 27' 51''$	9,6375587 co. ar.
To radius	90	10
So is sine $B \gamma$	$38^{\circ} 35' 51''$	<u>9,7950770</u>
To tang. $B \odot$		9,4326357

To find the sun's longitude (CLXVII).

As tan. $B \gamma$	$38^{\circ} 35' 51''$	0,0978784 co. ar.
To radius	90	10
So is cos. $\angle \gamma$	$23^{\circ} 27' 51''$	<u>9,9625158</u>
To cot. $\gamma \odot$	$41^{\circ} 1' 44''$	10,0603942

4. Three other Problems may be formed from the above four things concerned. Thus there may be given the sun's *longitude and right ascension*, the sun's *longitude and declination*, and the *obliquity of the ecliptic and declination*, to find the rest. These are left for the exercise of the learner.

5. It may be observed, that four triangles of the same dimensions as the last will be formed by the ecliptic, equator, and declination, at four different periods of the year; namely, about the 7th of February, the 1st of May, the 11th of August, and the 3d of November.

6. As both the longitude and right ascension are reckoned throughout the year from the first point of Aries; while the sun is ascending from Aries to Cancer in his first quadrant of the ecliptic, the given longitude $\gamma \odot$ is the hypotenuse in the triangle $\gamma \odot B$; γB is the right ascension, and $B \odot$ the declination North.

L 2

7. When

7. When the sun has passed the solstice ☊, and is descending toward Libra in his second quadrant, his longitude or distance from Aries being taken from 180° , the remainder $\sphericalangle$ ☉ becomes the hypotenuse; but the arc B $\sphericalangle$ found for the right ascension is only the supplement, and must therefore be taken from 180° . The declination is still North.

8. When the sun has passed the point Libra, and is descending toward Capricorn, he is said to be in his third quadrant, the longitude then reckoned from Aries will be greater than 180° ; the excess therefore above 180° , or the distance the sun is removed from Libra, will be the hypotenuse $\sphericalangle$ ☉, and the arc $\sphericalangle$ A found for the right ascension must be added to 180° to give the right ascension as reckoned from Aries. The declination is now South.

9. The sun having passed the solstice ☋ and being now ascending toward Aries in his fourth quadrant, the longitude is greater than 270 degrees, and must be therefore taken from 360 degrees to give the hypotenuse $\sphericalangle$ ☉: here $\sphericalangle$ A found by the proportion must be taken from 360 degrees, which will give the right ascension. The declination is still South.

In illustrating the following Problems by the Globe, it will be necessary first to rectify it, thus:

To rectify the globe for the Latitude, Zenith, and Noon.

For the LATITUDE. Raise or depress the pole until the given latitude on the brazen meridian coincide with the wooden horizon;

For

For the ZENITH. Screw the bevel edge of the nut belonging to the quadrant of altitude to the vertical point of the brazen meridian, which is the complement of the latitude from the elevated pole.

For NOON. Bring the sun's longitude or place in the ecliptic to the brazen meridian, and set the index of the hour circle to 12 o'clock.

Then is the globe rectified.

PROBLEM 4. FIG. 40.

Given the latitude of the place, and the sun's present declination.

Required the sun's altitude, and azimuth at six o'clock.

Exam. At London on the 21st of June, 1795, being the longest day, the sun's declination is $23^{\circ} 27' 51''$, and the latitude $51^{\circ} 32'$ North. *Required* the sun's altitude and azimuth at six o'clock in the morning or evening.

By the Globe.

1. *The globe being rectified*, turn it till the index points to six o'clock; move the quadrant of altitude till its edge cuts the first point of Cancer in the ecliptic (the sun's place); then the degrees cut between the horizon and sun's place will shew its altitude, and the degrees cut by the quadrant on the horizon will shew the azimuth, reckoning from the North.

By Projection.

2. Describe the primitive upon the plane of the meridian: let HR represent the horizon and ZN the prime

L 3

vertical;

vertical; make $RP = 51^{\circ} 32'$ the latitude N ; draw PS , the fix o'clock hour circle, and at right angles to it draw the equator EQ : describe the lesser circle mn at $23^{\circ} 27' 51''$ from the equator (cxiv) as the parallel of declination, and it will cut the fix o'clock hour circle PS in $\odot$, which is the sun's place at the given time.

Through $Z, \odot, N$, describe the azimuth circle $Z\odot N$ (xcv) cutting the horizon in A ; then the things given and required fall in either of the triangles $Z\odot P$ or $\odot A$, which are supplemental triangles one to the other. Thus RP = the latitude; ZP = the co-latitude; $A\odot$ = the altitude; $\odot Z$ = the co-altitude; $\odot$ = the declination; $\odot P$ = the co-declination; the $\angle \odot Z P$ measured by the arc AR = the azimuth, and the arc $\odot A$ = the co-azimuth.

3. *Let this Projection be compared with the corresponding position of the globe, as in the last article, HZN the primitive will represent the brazen meridian; HR the wooden horizon; RP the elevation of the pole; Z the vertical point or zenith; ZA the quadrant of altitude, and nm will represent a parallel of declination $23^{\circ} 27' 51''$ from the equator, which cuts the fix o'clock hour circle SP and the quadrant ZA in $\odot$.*

By Calculation.

4. In the right-angled spheric triangle $Z\odot P$ (right-angled at P)

$$\text{Given } \begin{cases} \text{the co-lat. } ZP & = 38^{\circ} 32', \\ \text{the co-declin } \odot P & = 66^{\circ} 32' 9''. \end{cases}$$

$$\text{Required } \begin{cases} \text{the co-altitude } Z\odot, \text{ and} \\ \text{the azimuth angle } \odot Z P \text{ or arc } AR. \end{cases}$$

Or,

Or, in the right angled spheric triangle $\nabla \odot A$ (right-angled at A)

Given $\left\{ \begin{array}{l} \text{the lat. or angle } \odot \nabla A = 51^{\circ} 32', \\ \text{the declin. } \nabla \odot = 23^{\circ} 27' 51''. \end{array} \right.$

Required $\left\{ \begin{array}{l} \text{the altitude } A \odot, \\ \text{the co-azimuth } \nabla A. \end{array} \right.$

To find the altitude $\odot A$ (CXLII).

As radius	90°	co-ar.
To fine declin.	$= 23^{\circ} 27' 51''$	9,6000745
So is fine latitude	$= 51 \quad 32$	9,8937452
To fine altitude	$= 18 \quad 9 \quad 55$	9,4938197

To find the azimuth angle $\odot Z P$, or arc AR (CLXIX).

As tan. $\odot B$ co-decl.	$= 66^{\circ} 32' 9''$	9,6375587	co-ar.
To radius	90	10	
So is fine $Z P$ co-lat.	$= 38 \quad 28$	9,7938317	
To cot $\odot Z P$ azim.	$= 74 \quad 53 \quad 22$	9,4313904	

5. If the foregoing question were changed from the longest day at London to the shortest, the parallel of South declination $c d$ would cut the 6 o'clock hour circle below the horizon: now as the triangles $\nabla A \odot$ and $\nabla a \odot$ are congruous, the *depression* $a \odot$ below the horizon on the shortest day at 6 o'clock will be equal to the *altitude* $A \odot$ at the same hour on the longest day; and the azimuth will be also equal if estimated from the South.

6. Thus on the 21st of June at London, at 6 o'clock in the morning, the sun will bear $74^{\circ} 53' 22''$ East from

the *North point*, and at the same hour in the evening it will have the same azimuth West, from the North point; but on the 21st of December at the same hours the sun will bear $74^{\circ} 53' 22''$ East and West from the *South point*.

7. By this Problem it is evident, that as the declination increases, the altitude at 6 increases, and the azimuth lessens; but the contrary happens when the declination is diminishing.

It is also evident, that when the sun has no declination the altitude at 6 o'clock will be nothing, for the sun will be then in the horizon at Aries or Libra, and the azimuth ($\propto$ R) being then 90 degrees, the sun will be due East in the morning, and due West in the evening. Thus on the days of the equinoxes the sun rises and sets at six, in the East and West points of the horizon.

PROBLEM 5. FIG. 41.

Given the latitude of the place, and the sun's declination.

Required the altitude, and the hour when the sun is due East or West.

Exam. At London on June 21, 1795, what is the sun's altitude, and the hour when he is due East or West?

By the Globe.

1. The globe being rectified, make the quadrant of altitude coincide with the East or West points on the wooden horizon. Turn the globe till the sun's place in the

the ecliptic touches the quadrant, then the index will point to the hour, and the arc of the quadrant which is between the sun's place and the horizon will shew his altitude.

By Projection.

2. Let the primitive circle HZRN be made to represent the meridian of London; draw the horizon HR and the prime vertical ZN; make $RP = 51^{\circ} 32'$ the latitude; draw the 6 o'clock hour circle PS and the equator EQ; let the parallel of declination nm be drawn $= 23^{\circ} 27' 51''$ from EQ (cxiv) which will cut the prime vertical in $\odot$, and through P, $\odot$, S, describe the hour circle P $\odot$ S (xcv).

Here the primitive represents the brazen meridian on the globe; HR the wooden horizon, and Z $\odot$ the quadrant of altitude.

Observe that the things concerned in the problem fall in either of the triangles PZ $\odot$ or $\odot$ A $\odot$: $\odot$ $\odot$ = the altitude and $\angle$ ZP $\odot$ = the horary angle or hour from noon.

By Calculation.

3. In the spheric triangle PZ $\odot$, right-angled at Z;

Given $\begin{cases} \text{the co-lat. PZ} = 38^{\circ} 28'. \\ \text{the co-declin. P } \odot = 66^{\circ} 32' 9''. \end{cases}$

Required $\begin{cases} \text{the co-altitude Z } \odot, \\ \text{the hour from noon } \angle \text{ ZP } \odot. \end{cases}$

To find the co-altitude $Z \odot$ (CLVI).

As cos. co-lat. PZ	$38^{\circ} 28'$	0,1062548 co. ar.
To radius	90°	10
So is cos. co-dec. $P \odot$	$66^{\circ} 32' 9''$	<u>9,6000745</u>
To cos. co-alt. $Z \odot$	$59^{\circ} 25' 59''$	9,7063293

Hence $30^{\circ} 34' 1''$ is the sun's altitude when due East or West the 21st of June, 1795.

To find the hour from noon, $\angle ZP \odot$ (CLVII).

As radius		co. ar.
To tan. co-lat. PZ	$38^{\circ} 28'$	9,9000865
So is cot. co-decl. $P \odot$	$66^{\circ} 32' 9''$	<u>9,6375587</u>
To cos. hour from noon $\angle ZP \odot$	$69^{\circ} 49' 36''$	9,5376452

Which $69^{\circ} 49' 36''$ converted into time gives 4 h. $39' 18'' 24'''$, and shews the time before and after noon, when the sun will be due East or West on the 21st of June, 1795.

The above Problem may be performed by the triangle $\triangle A \odot$, right-angled at A; for here are given the latitude $\angle A \odot$ and the decl. $A \odot$, whence the altitude $\odot$ is found by Art. CXLII, and the hour from 6 o'clock A $\odot$ by Art. CLXIII.

4. If it be required to work this Problem for the shortest day, draw the other circle of declination cd ; then the triangle $\triangle a \odot$ being congruous to $\triangle A \odot$, by the same operation the sun's depression when due East or West is found, as well as the hour, which will be before 6 in the morning and after 6 in the evening, by as much as the contrary was found above,

5. By

5. By this Problem it is easy to see, that when the latitude of the place and the sun's declination have both the same names, then the greater the declination and latitude, the greater the sun's altitude when due East or West, and the longer the time from 6 o'clock: and when they have contrary names the same things take place, but with this difference, that when the latitude and declination have the same names their increase lengthens the days, whereas when they have contrary names their increase shortens the days.

P R O B L E M 6. FIG. 42.

Given the latitude of the place, and the sun's declination.

Required his amplitude, and ascensional difference.

By this interesting Problem the length of the day and night is determined, for *the ascensional difference converted into time shews how long before or after 6 o'clock the sun rises or sets.*

Exam. *At London on the 21st day of June, 1795, how far from the East and West does the sun rise and set; and at what time?*

By the Globe.

1. The globe being rectified, bring the sun's place in the ecliptic to the horizon; then the amplitude is that degree of the horizon opposite the sun's place, reckoning from the East point in the morning and from the West in the evening.

The ascensional difference is that arc of the equator below the horizon intercepted between it and the solstitial colure. Or,

The

The sun's right ascension on that day being a quadrant, and its oblique ascension the degree of the equator cut by the horizon: the ascensional difference is the difference between the right and oblique ascension.

By thus turning the globe the index will point to the time of the sun's rising and setting.

By Projection.

2. Let the primitive circle represent the meridian of the place, the diameter HR the horizon, and Z N the nadir. From R the North of the horizon, make $RP = 51^{\circ} 32'$ for the latitude. Draw the axis or 6 o'clock hour circle PS, and EQ the equator at right angles to it; make En and Qm $= 23^{\circ} 27' 51''$ the declination, and describe nm (cxiv) cutting the horizon in ☉ the place of the sun at its rising and setting, through which place describe the hour circle P ☉ S (xcv).

3. Now it is evident that on the 21st of June the apparent circle which the sun makes round the earth, is a parallel circle, represented by nm in the Projection; at n the sun is in the meridian, and describes the quadrant nx in six hours.

4. The question therefore (of finding the sun's rising and setting) is to measure the arc x ☉ of the lesser circle, and convert it into time, which will shew how long the sun is describing the space x ☉, and this being added or subtracted from 6 will give the time of sun-rise or sun-set.

5. But this arc x ☉ of the small circle nm contains the same number of degrees as its corresponding arc

☉ A

φ A of the great circle or equator E Q, and therefore the things concerned in the Problem fall in the triangle $\varphi \odot A$, right-angled at A; φ A is the ascensional difference, and $\varphi \odot$ the amplitude; these are the two required arcs, and they may be measured on the scale of semi-tangents (LXXIII).

By Calculation.

In the spheric triangle $\varphi \odot A$, right-angled at A,

Given $\left\{ \begin{array}{l} \text{the sun's decl. } A \odot = 23^{\circ} 27' 51'', \\ \text{co-lat. } \angle A \varphi \odot = 38^{\circ} 28', \\ \text{(for } \angle A \odot \varphi = \text{arc QR} = \text{arc ZP the co-lat.)} \end{array} \right.$

Required $\left\{ \begin{array}{l} \text{the amplitude } \varphi \odot, \text{ and} \\ \text{the ascensional difference } \varphi A. \end{array} \right.$

6. To find the amplitude $\varphi \odot$ (CXLII).

As sine co-lat. $\angle A \varphi \odot = 38^{\circ} 28'$	0,2061683
To sine sun's decl. $A \odot \quad 23 \quad 27 \quad 51$	9,6000745
So is radius 90	<u>10</u>
To sine amplit. $\varphi \odot \quad 39 \quad 48 \quad 3$	9,8062628

7. To find the ascensional difference $\varphi \odot$ (CLXIII.)

As radius 90°	
To tan. decl. $A \odot \quad 23^{\circ} 27' 51''$	9,6375587
So is cot. co-lat. $\angle A \varphi \odot \quad 38 \quad 28$	<u>10,0999135</u>
To sine ascen. diff. $\varphi A \quad 33 \quad 7 \quad 2$	9,7374722

Here the ascensional diff. $\varphi A \quad 33^{\circ} 7' 2''$ turned into time gives 2h. 12m. 28s. 8'', the time which the sun rises before and sets after 6 at London on the 21st of June, 1795, which is the longest day.

8. Now

8. Now as the sun apparently describes the parallel of declination nm in 12 hours, being at n when it is noon and at m when it is midnight, therefore the time of passing from $\odot$ to m is half the night, that is, from sun-set to midnight; and round from m to $\odot$ is the time from midnight to sun-rise; hence $m \odot$ doubled is the length of the night, and $n \odot$ doubled the length of the day at the above given time.

Then to and from	6h.	0'	0"	0'''
Add and subtract the ascen. diff.	2	12	28	8
Sum gives $n \odot$ half day	8	12	28	8
Difference gives $\odot m$ half night	3	47	31	52
Longest day at London, 1795,	16	24	56	16
Shortest day, ditto	7	35	3	44

9. If in the foregoing figure, rs be supposed a parallel of declination as far South as nm is North, then the hour circle PBS passing through $\odot$, the place of the sun at his rising or setting will form a triangle $\gamma \odot B$ congruous to the triangle $\gamma \odot A$, where the amplitude $\gamma \odot$ is Southward of the East and West points, and the ascensional difference γB converted into time shews how long after fix in the morning and before fix in the afternoon the sun rises or sets on the 21st of December, the shortest day.

10. It therefore appears that when the latitude and declination are both North or both South, the sun rises before and sets after fix, but when they are of different names the sun rises after and sets before fix.

11. It

11. It is also evident, that when the latitude and declination have the same name, the difference between the right ascension and the ascensional difference is the oblique ascension, and their sum is the oblique descension. But when they are of contrary names, their sum is the oblique ascension, and their difference is the oblique descension.

12. From a due consideration of this Problem, it is clear that when the declination is equal to the co-latitude of any place (which can only happen within the polar circles) then the parallel of declination will not cut the horizon, and consequently the sun will not set in those places during the time his declination exceeds or is equal to the co-latitude. This effect takes place also with regard to the stars, but it is to be understood only, when the sun or stars are in the same hemisphere with the given place; for when they are in the contrary hemispheres, if the co-latitude of the place does not exceed the declination of the celestial objects under consideration, then will these objects remain always below the horizon, and consequently be invisible.

Thus in the latitude of London, all the stars in the northern hemisphere, whose declinations are above $38^{\circ} 28'$ (the co-latitude) never are seen to set, but the stars of the same declination in the southern hemisphere are never visible at London.

Thus also when the sun's declination is greatest North, it always appears above the horizon within the arctic circle, and according as the declination decreases this effect decreases from the arctic to the North pole; but when the declination is South, the sun by the same rule

is continually visible within the antarctic, and invisible within the arctic circle.

13. The following short rule for finding the time of the sun's rising or setting is deduced from Art. 7 of this Problem, viz. *The sum of the tangents of the latitude and declination, rejecting 10, will give the sine of the ascensional difference, which turned into time, and added to or subtracted from 6, will shew the hour of the sun's rising or setting.*

It may be observed, that the time thus found differs about 9' from that in the almanacs; this is an allowance for refraction, which makes the apparent day about 9 minutes longer than the astronomical day.

As the time of sun rising or setting is not given in the almanacs in a lower denomination than minutes, notice cannot be there taken of the small decrease in our longest day and increase in our shortest, which is about 6 seconds of time in 100 years.

14. This change in the length of the days and nights is produced by the diminution of the obliquity of the ecliptic, for the various observations which have been made to determine it for about 2000 years past prove the gradual decrease of this angle, but all do not agree as to the quantity, some making it 50" in a hundred years, and others more; there is reason therefore to think that the diminution is variable. If it should continue to decrease at the rate of half a second annually, the ecliptic would coincide with the equator in about 169 thousand years, and then of course an equality of day and night, and perhaps of seasons, would universally prevail.

P R O-

PROBLEM 7. FIG. 43.

Given the latitude of the place, and the sun's declination.

Required the time when the twilight begins and ends.

Exam. *At what time does the twilight begin and end at London on the 1st of May, 1795?*

By the Globe.

1. The globe being rectified, turn it, and move the quadrant of altitude till the sun's place in the ecliptic stands against 18 degrees on the quadrant of altitude, below the horizon; then will the index shew the beginning or end of twilight, that is, the beginning in the morning when those points meet in the East, or the end in the evening when they meet in the West.

By Projection.

2. Draw ZRNH to represent the meridian of the place, HR the horizon, ZN the prime vertical, and ts a small circle parallel to the horizon described at 18 degrees below it (cxiv). Lay off the latitude RP, and draw the axis PS and the equator EQ, then describe the parallel of declination nm for the first of May, and where nm cuts ts in $\odot$ is the sun's place at the beginning or end of twilight: through $\odot$ describe the vertical circle Z $\odot$ N and the hour circle P $\odot$ S (xcv), then the angle ZP $\odot$ being measured (cv) will give the time before or after noon when the twilight begins or ends.

It is obvious that as the sun is at n in the meridian at noon on the first of May, its apparent diurnal motion is represented by the small circle nm , and the arc $n\odot$ is the distance to be measured, which shews the time from noon to the end of twilight: now the arc $n\odot$ is

M

measured

measured by the arc Ex , which is the same as the angle $ZP\odot$ (xix).

By Calculation.

3. In the oblique spheric triangle $ZP\odot$:

Given the co-lat. $ZP = 38^\circ 28'$,

the polar dist. $P\odot = 74^\circ 50' 53''$ (the comp. of decl.)

Required the angle $ZP\odot$ the hour from noon.

The following solution is the same as shewn in Art. CLXXXII.

$$\begin{aligned} PZ &= 38^\circ 28' \\ P\odot &= 74^\circ 50' 53'' \\ Z\odot &= 108 \end{aligned}$$

$$2) \begin{array}{r} 221 \\ 18 \\ 53 \end{array}$$

$$\text{half sum } \begin{array}{r} 110 \\ 39 \\ 26\frac{1}{2} \end{array}$$

$$\frac{1}{2} \text{ sum} - PZ = \begin{array}{r} 72 \\ 11 \\ 26\frac{1}{2} \end{array} \text{ fine} \quad 9.9786733$$

$$\frac{1}{2} \text{ sum} - P\odot = \begin{array}{r} 35 \\ 48 \\ 33\frac{1}{2} \end{array} \text{ fine} \quad 9.7672222$$

$$\text{Co ar. s. } 38^\circ 28' \quad 0.2061683$$

$$\text{Co-ar. s. } 74^\circ 50' 53'' \quad 0.0153665$$

$$\hline 19.9674303$$

$$\text{Sine } 74^\circ 24' 26\frac{1}{2} \quad 9.9837151$$

2

$$15) \begin{array}{r} 148 \\ 48 \\ 53 \end{array} = \angle ZP\odot$$

$$9 \ 52$$

$$3 \ 12$$

$$3 \ 32$$

$$\angle ZP\odot \text{ reduced to time} = 9^h.55' 15'' 32''', \text{ either before or}$$

FIG. 44.

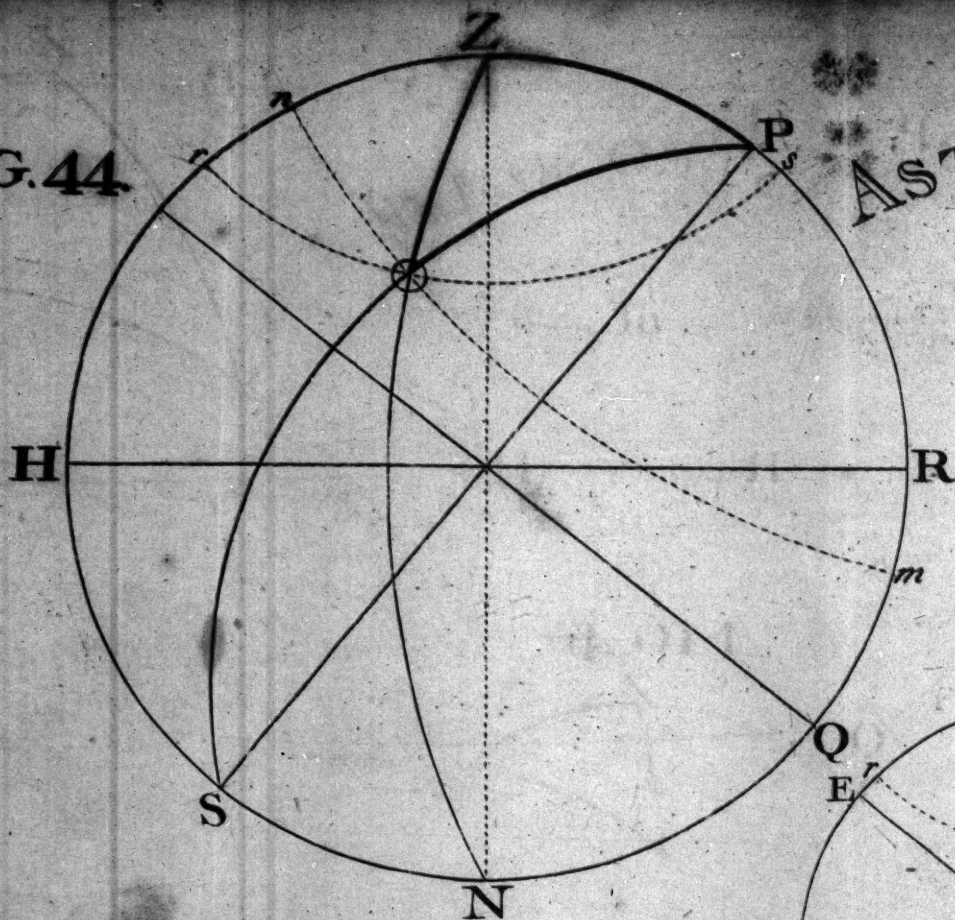


FIG. 45.

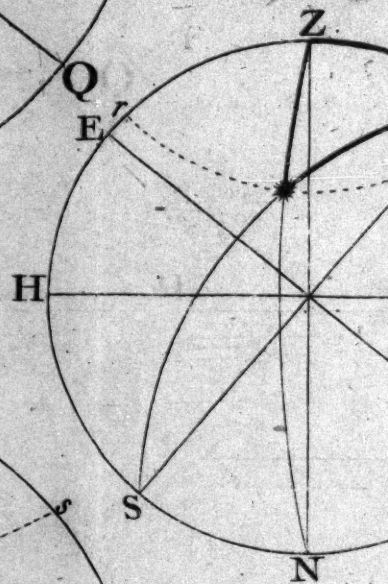
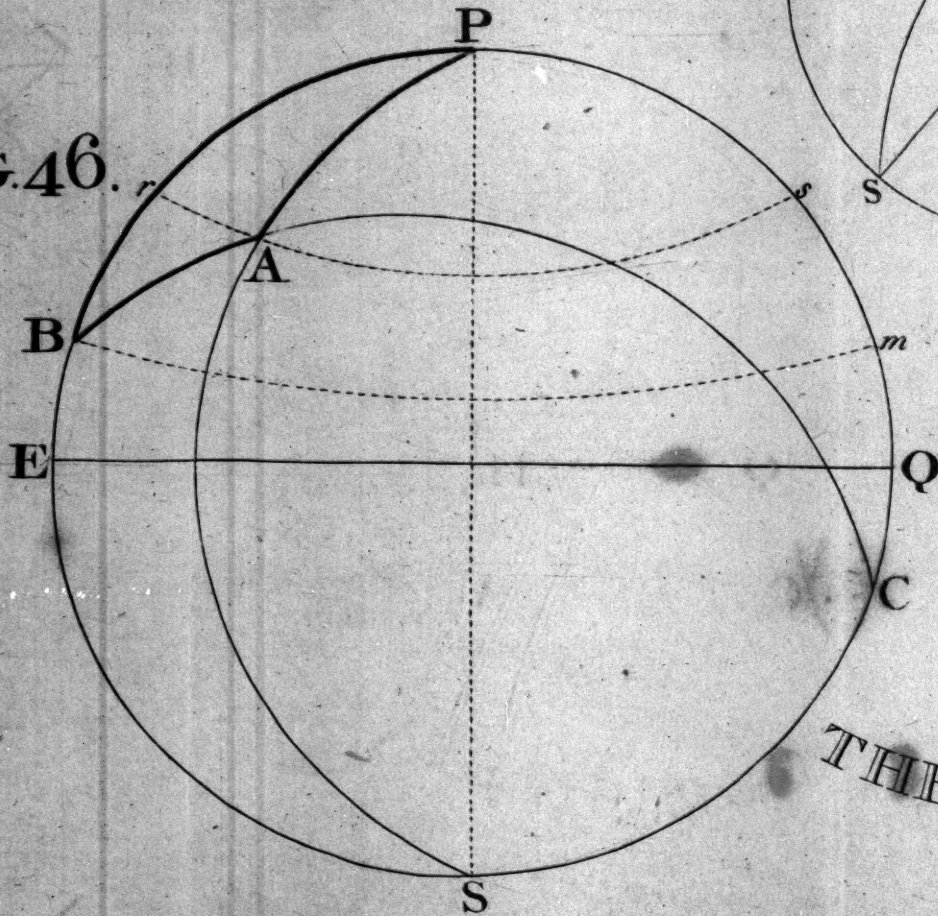


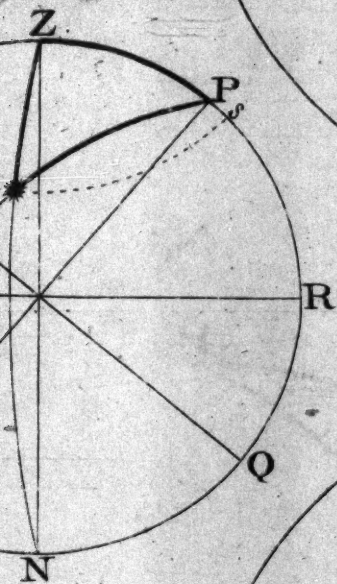
FIG. 46.



RONOMY

of

G. 45.



SPHERE

FIG. 47.

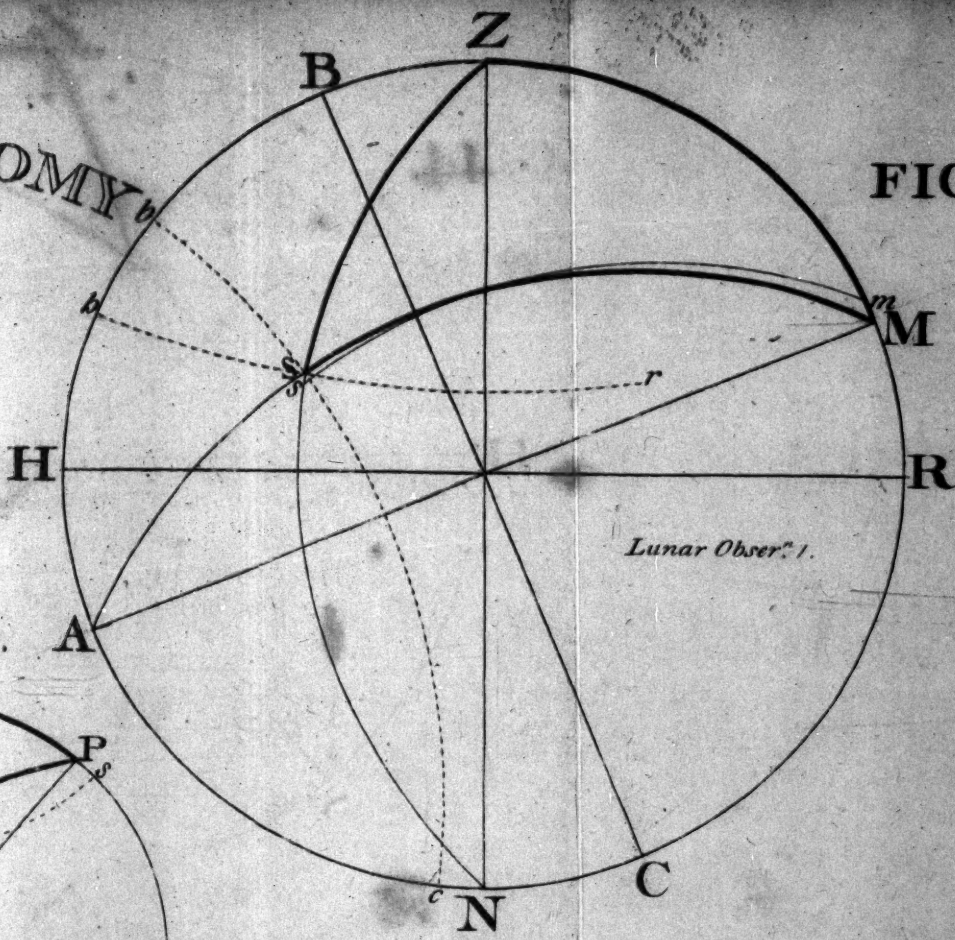
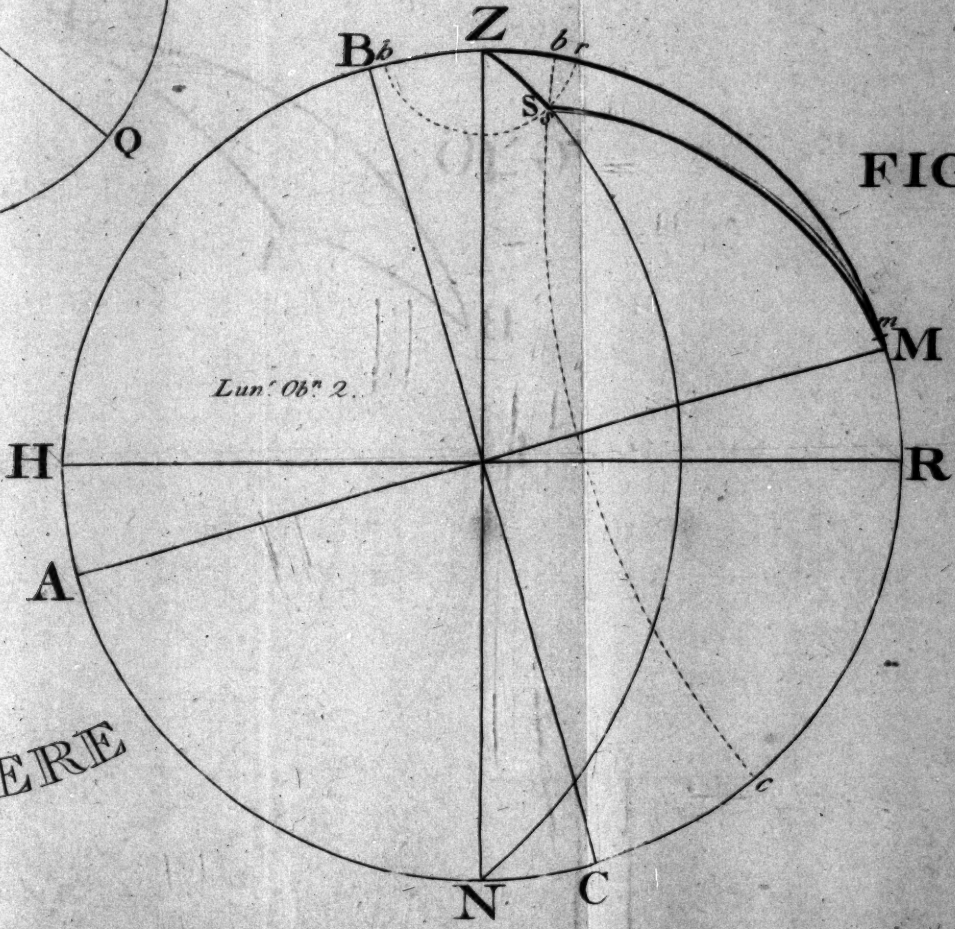


FIG. 48.



or after noon. Thus the twilight begins at 2 h. 4' 44" in the morning, and ends at 9 h. 55' 15" in the evening at London on the first of May, 1795.

4. It is evident by the figure under consideration, that when the declination becomes greater than the difference between the co-latitude and 18 degrees, the parallel of declination nm will not cut the parallel circle ts 18 degrees below the horizon, but m will be above s , and therefore at that time there will be no night in the given latitude, but the twilight will continue from sun setting to sun rising. Thus at London, when the declination is above $20^{\circ} 28'$ ($38^{\circ} 28' - 18^{\circ}$) there is no total darkness: this takes place about the 28th of May, and continues to about the 20th of July.

The above rule holds for any other latitude; that is, when the sun's declination added to 18° is more than the co-latitude of the place there is no night.

PROBLEM 8. FIG. 44.

Given the sun's altitude and declination, with the latitude of the place.

Required the hour of the day, and the sun's azimuth.

Exam. At London on the 21st of June, 1795, the sun's altitude was observed to be $46^{\circ} 20'$. What was the hour, and the sun's azimuth?

By the Globe.

1. The globe being rectified, turn it, and move the quadrant of altitude until the sun's place in the ecliptic coincides with the complement of the altitude on the

M 2

quadrant;

quadrant; then will the index shew the hour, and the quadrant will cut in the horizon the degree of azimuth, reckoned from the North.

By Projection.

2. Let the primitive circle $ZRNH$ represent the meridian of London, HR the horizon and ZN the prime vertical; make $RP = 51^{\circ} 32'$, the height of the pole at London; draw the axis PS and the equator EQ ; lay off En and $Qm = 23^{\circ} 27' 51''$, the sun's declination, and HR and $Rs = 46^{\circ} 20'$ the altitude. Draw the small circles nm , and rs , (cxiv). Where these two parallel circles intersect in $\odot$ is the sun's place at that time.

Through $Z, \odot, N$, describe the azimuth circle $Z\odot N$, and through $P, \odot, S$, describe the hour circle $P\odot S$ (xcv): then the angle $\odot PZ$ being measured (cv) will give the hour from noon, and the angle $Z\odot P$ will give the azimuth.

It is plain by the figure that the arc $n\odot$ represents the path described by the sun from noon, and that this is measured by the arc of its corresponding great circle Ex , which is the measure of the angle EPx or $ZP\odot$ (xix).

It is also obvious that the arc yR measures the distance of the azimuth circle $Z\odot N$ from R the North point of the horizon, and that this arc is the measure of the angle yZR or $\odot ZP$.

By Calculation.

In the oblique angled spheric triangle $P\odot Z$ the three sides are given to find the angles at P and Z . Thus,

I

Given

Given the co-lat. $ZP = 38^{\circ} 28'$,
 the co-alt. or zen. dist. $Z\odot = 43 40$,
 the co-decl. or pol. dist. $\odot P = 66 32 9''$.

Required the horary angle $\odot PZ$, and
 the azimuth angle $Z\odot P$.

Both parts of this useful Problem are performed by
 Art. LXXXII.

3. To find the hour from noon, $\angle \odot PZ$.

$$\begin{array}{r}
 38^{\circ} 28' \\
 43 40 \\
 66 32 9'' \\
 \hline
 2) 148 40 9 \\
 \hline
 \frac{1}{2} \text{ sum } 74 20 4\frac{1}{2} \\
 \hline
 \frac{1}{2} \text{ sum} - ZP = 35 52 4\frac{1}{2} \text{ fine} \quad 9.7678374 \\
 \frac{1}{2} \text{ sum} - \odot P = 7 47 55\frac{1}{2} \text{ fine} \quad 9.1325558 \\
 \hline
 \text{Co-ar. s. } 38^{\circ} 28' \quad 0.2061683 \\
 \text{Co-ar. s. } 66 32 9'' \quad 0.0374842 \\
 \hline
 2) 19.1440352 \\
 \hline
 \text{Sine } 21^{\circ} 55' 1'' \frac{1}{2} \quad 9.5720176 \\
 \hline
 \end{array}$$

$43 50 3 =$ the angle from
 noon, which converted to time is 2 h. 55' 26" 12".

Hence the observation was made either at 9 h. 4' 33" in the
 morning, or at 2 h. 55' 26" in the afternoon.

4. To find the azimuth, $\angle \odot ZP$.

If the hour be previously found, the azimuth may be
 thence determined by the proportion between opposite

M 3

fides

sides and angles, or the azimuth being first found, the hour may be thence determined; but both are determined the same way in the first instance. Thus the half sum of the sides being found as above, subtract from it the two sides which contain the angle $\odot Z P$, and the remainders will be,

$$\text{1st rem. } 35^{\circ} 52' 4'' \frac{1}{2} \text{ fine} \quad 9.7678374$$

$$\text{2d rem. } 30 40 4 \frac{1}{2} \text{ fine} \quad 9.7076224$$

$$\text{Co-ar. s. } 38^{\circ} 28' \quad 0.2061683$$

$$\text{Co-ar. s. } 43 40 \quad 0.1608604$$

$$2) 19.8424885$$

$$\text{Sine } 56^{\circ} 31' 39'' \quad 9.9212445$$

$$\underline{\quad\quad\quad} \quad 113 \quad 3 \quad 18 = \text{the azimuth sought,}$$

reckoning from North.

If the latitude and declination had been of contrary names, the same kind of operation would have been used to find the things required; but the side $\odot P$ would extend beyond the equator $E Q$, and being therefore obtuse, the declination must be added to 90° , whereas when the latitude and declination are of like names, the latter is to be subtracted from 90° .

From the five things concerned in this problem, namely, the *latitude*, *altitude*, *declination*, *azimuth*, and *hour*, several other useful problems may be formed; for any three out of the five being given, the rest may be found by oblique spheric trigonometry.

The following three problems are given with only general instructions, the numerical solutions being left for the exercise of the learner.

P R O-

PROBLEM 9. FIG. 44.

Given the sun's altitude, the hour of the day, and the latitude of the place.

Required the azimuth and declination.

Exam If the altitude of the sun was found to be $46^{\circ} 20'$ in the latitude of London, and the time 2 h. 55' 26", what was the sun's azimuth, and declination?

In this problem the data are the side Z P or co-lat. the side Z $\odot$ or co-alt. and the opposite $\angle$ Z P $\odot$ or hour; and the quæsitæ are P $\odot$ or the co-decl. and $\odot$ Z P or the azimuth.

Thus in the oblique angled spheric triangle there are given two sides and an opposite angle; the figure may therefore be projected according to Art. CXXXII, and the computation made as in Art. CLXXVII or CLXXXIX, which will give the azimuth $113^{\circ} 3' 18''$, and the declination $23^{\circ} 27' 51''$.

PROBLEM 10. FIG. 44.

Given the latitude, azimuth and hour.

Required the altitude and declination.

Exam. At London when the sun's azimuth is $113^{\circ} 3' 18''$, and the hour 2 h. 55' 26", what is the sun's altitude and declination?

Here the three things given are the side Z P or co-lat. the $\angle$ at Z or azimuth, and the angle at P or hour: hence two angles and an included side are given to find

the other two sides, viz. $Z \odot$ the co-alt. and $P \odot$ the co-decl. the figure may therefore be projected by Art. cxxxv, and the numerical solution performed by Case 4, p. 80 or 91: thus the altitude is found $= 46^{\circ} 20'$, the declination $= 23^{\circ} 27' 51''$.

PROBLEM 11. FIG. 44.

Given the latitude of the place, the sun's declination, and the hour of the day.

Required the sun's altitude, and azimuth.

Exam. *What is the sun's altitude, and azimuth at London on the 21st of June, 1795, at 2 h. 55 m. 26 s.?*

Here the three things given are the side ZP or co-lat. the side $P \odot$ or co-decl. and the angle $ZP \odot$ or hour; hence in the oblique angled spheric triangle two sides and an opposite angle are given to find the rest; the figure therefore may be projected by Art. cxxxii, and the calculation performed by Case 1, p. 74 or 87, by which the sun's altitude will be found $46^{\circ} 20'$, and its azimuth $113^{\circ} 3' 18''$.

Several other variations may be made from the five things here concerned, but these given are sufficient to shew how such problems are performed.

PROBLEM 12. FIG. 45.

Given the latitude of the place, time of the year, and the altitude of a known fixed star.

Required the hour of the night when the observation was made.

Exam.

Exam. Some time of the night on the 1st of May, 1795, the star Arcturus was observed at London to be $45^{\circ} 20'$ above the horizon, at what hour was the observation made.

Here the declination and ascension of Arcturus being found in a Table of the Fixed Stars $\ast = 20^{\circ} 17' 16''$, the problem is performed on the same principles as problem 8th, for the three sides are given, viz. $Z \ast$ = the co-altitude or zenith distance, $P \ast$ = the co-declination or polar distance, and ZP = the co-latitude; to find the angle $ZP\ast$. Now the measure of this angle is the arc Ex , which also measures the arc $n\ast$, the path that the star must describe before it comes to the meridian: when this arc is known, find the time of the star's culminating by the following rule.

2. *From the right ascension of the given star subtract the sun's right ascension for the given day, the difference will be the time of the star's culminating nearly. Say as 24 h. is to the daily change of the sun's right ascension, so is the time of culminating, nearly, to a fourth number; which number being subtracted from the time of culminating, nearly, will give the true time of the star's culminating.*

Observe,

$\ast$ The Table from whence the following declinations and right ascensions of the fixed stars are taken is for the year 1790, without any correction for their annual variation since that time. The learner should however understand the reason of those variations, and apply the corrections when necessary.

By the precession of the equinoxes the fixed stars are continually altering their declinations, right ascensions, and longitudes, but they always keep the same latitudes. The longitude increases about $50\frac{1}{3}$ seconds regularly every year, but the variations of right ascension and declination are not regular; some stars which had once North declination have now South, and vice versa. The necessary corrections for annual variation are generally inserted in the Tables.

Observe, if the sun's right ascension be greatest, to add 24 hours to the star's right ascension.

The time of the star's culminating being thus found, subtract the angle ZP^* (turned into time) from it, and the remainder will shew the hour when the observation was made.

3. This problem being performed both by the globe and by projection precisely in the same manner as problem 8th; the numerical solution is also the same. Thus, by Art. CLXXXII,

$$\begin{array}{rcl}
 Z^* \text{ zen. dist.} & = & 44^\circ 40' \\
 P^* \text{ polar dist.} & = & 69 \ 42 \ 44'' \\
 ZP \text{ co-lat.} & = & 38 \ 28 \\
 \hline
 & 2) & 152 \ 50 \ 44 \\
 \hline
 \frac{1}{2} \text{ sum} & & 76 \ 25 \ 22 \\
 \hline
 1^{\text{st}} \text{ rem.} & 37 \ 57 \ 22 & \text{fine} \quad 9,7889158 \\
 \hline
 2^{\text{d}} \text{ rem.} & 6 \ 42 \ 38 & \text{fine} \quad 9,0676425 \\
 \hline
 & & \text{Co. ar. pol. dist. } 0,0278143 \\
 & & \text{Co-ar. co-lat. } 0,2061683 \\
 & & \hline
 & & 2) 19,0905409 \\
 & & \hline
 & & \text{Sine } 20^\circ 32' 48'' = 9,5452704 \\
 & & \quad \quad \quad 2 \\
 & & \hline
 \text{Angle } * PZ & = & 41 \ 5 \ 36 \\
 & & \hline
 \end{array}$$

This $41^\circ 5' 36''$ turned into time gives 2 h. 44 m. 22 s. for the time which must elapse before the star comes to the

the meridian. Now at the time of observation, May 1, 1795,

	h.	m.	s.
The sun's right ascension is	-	-	2 34 23
The star's right ascension is	-	-	14 6 5
Diff. of right ascension, the time of cul- minating nearly	-	-	11 31 42
The daily change of the sun's right ascen- sion is $3' 49''$: then as $24 : 3' 49'' :: 11^{\circ}$ $31' 42'' : 1' 41''$	-	-	1 41
True time of the star's culminating	11	30	1
Time the star has to come to the meridian	2	44	22
The difference gives the hour of observation	8	45	39

If the observed star be in the western part of the horizon, or past the meridian, addition must be used, which will give the time of observation 14 h. 14 m. 23 s.

4. From the four things here concerned, namely, the *altitude, declination, latitude, and hour*, three other problems may be formed; for any three of the above being given, the fourth is found by the rules of oblique trigonometry.

PROBLEM 13. FIG. 46.

Given the right ascensions and declinations of two known fixed stars.

Required their distance asunder.

Exam. What is the distance between the fixed stars Aldebaran in Taurus, and Algol in Caput Medusæ, the right ascension of the former being 4 h. 23 m. 53 s. and its declination

tion $16^{\circ} 4' 25''$ N. the right ascension of the latter being 2 h. 54 m. 34 s. and its declination $40^{\circ} 8' 3''$ N.

1. This problem is performed on the globe by laying the quadrant of altitude on the two given stars, which shews the number of degrees they are asunder, for the quadrant always measures the nearest distance on the surface of a sphere, and therefore represents the arc of a great circle.

2. By Projection.

The difference between the two given right ascensions being turned into degrees will $= 22^{\circ} 19' 45''$; this is the angle the two meridian circles passing through Aldebaran and Algol make with each other at the pole, this angle being measured by that arc of the equator which is the difference of their right ascensions (xix).

Let the primitive circle be drawn to represent the meridian or circle of right ascension, which passes thro' Aldebaran,

Draw EQ for the equator, PS the axis, P being the North pole and S the South pole.

Describe the circle of right ascension PAS to make the angle BPA $= 22^{\circ} 19' 45''$, the difference between the two given right ascensions (cix).

Describe the parallels of declination Bm and rs, at the given distances $16^{\circ} 4' 35''$ and $40^{\circ} 8' 3''$ (cxiv), and the intersections B, of Aldebaran's declination, and A, of Algol's, with their respective circles of right ascension, will be the places or positions of those stars.

Then

Then draw a great circle BAC through B and A (cx), and the intercepted arc BA measured by Art. ciii shews the distance between those two stars.

3. By Calculation.

In the oblique angled spheric triangle PAB,

Given the co-declination of Aldebaran PB = $73^{\circ} 55' 35''$
 the co-declination of Algol PA = $49^{\circ} 51' 57''$
 the diff. of right ascension $\angle APB = 22^{\circ} 19' 45''$.

Required the side BA, their distance asunder.

Here two sides and the included angle being given, the other side is found by Case 2, pages 76, 89, or 90.

Thus by the method in page 90;

Side PB	= $73^{\circ} 55' 35''$	fine	9,9826813
Side PA	= $49^{\circ} 51' 57''$	fine	9,8833986
$\frac{1}{2}$ incl. $\angle APB$	= $11^{\circ} 9' 52\frac{1}{2}''$	double s.	18,5739260
			2)38,4400059
$\frac{1}{2}$ diff. of sides	$12^{\circ} 1' 49''$	fine	19,2200029
			9,3189572
Tan.	$38^{\circ} 31' 42''$		
			9,9010457
Sine	$38^{\circ} 31' 42''$		
			9,7944194
$\frac{1}{2}$ sum — fine 38° , &c.	= $15^{\circ} 27' 7''$		
			9,4255835
			2
			30 54 14

the distance between the stars Aldebaran and Algol.

4. By

4. By the same method the distance between any two fixed stars may be found when their latitudes and longitudes are given; only instead of latitude read declination, and instead of longitude use right ascension.

5. By the same rule the distance between any two places on the terrestrial globe may be found, when the latitude and longitude of each are given. Thus the co-latitudes are two sides of an oblique angled spheric triangle meeting at the pole, and the difference of longitude is that arc of the equator which measures the included angle.

6. Thus Spheric Trigonometry is applied to many useful problems in Geography and Navigation, but the learner who understands what has been hitherto delivered will easily comprehend such applications, as well as various other problems of Astronomy which are to be found in books upon this subject.

SECTION IX.

UPON THE LATITUDE.

VARIOUS METHODS OF FINDING THE LATITUDE.

1. THE most practical method of finding the latitude at sea is from the meridian altitude of the sun, or other celestial object, whose declination is known, and thence finding the distance from the zenith to the equinoctial, which is the latitude of the place; but as it frequently happens that the heavenly bodies cannot be observed at
the

the time of culminating, recourse must be had to other methods, a variety of which will be given in this Section.

2. In the foregoing problems, as well as in most of those that follow, the altitudes are assumed as true; but it may be proper here to shew how such are found, that is, how the apparent altitude is cleared from the effects of *Refraction, Dip of the Horizon, Semi-diameter and Parallax*; and next to shew how the sun's declination is found for any given hour, or longitude, from the declination at noon as given in the Nautical Almanac.

In order to clear the altitude, the following rules must be observed.

3. *From the apparent altitude subtract the refraction and the dip, for these two things always make the heavenly bodies appear higher than they really are.*

If the upper limb of the sun or moon be observed, the semi-diameter must be subtracted, but added, if the lower limb be observed.

The parallax must be always added, for it makes those bodies which it affects appear lower than they really are.

Neither parallax nor semidiameter occurs in the altitude of a fixed star.

Example.

On the first of June, 1795, the altitude of the sun's lower limb was observed to be $60^{\circ} 25' 27''$, the eye being 16 feet above the horizon. What was the sun's true central altitude?

Appar.

Appar. alt. sun's lower limb		60° 25' 27"
Refraction (per req. Tab.)	- 0' 32"	
Dip. of Hor. (per ditto)	- 3 49	
		4 11
		60 21 16
Sun's sem. diam. (Naut. Alm.)	+ 15 49	
parallax (req. Tab.)	+ 4	
		+ 15 53
True alt. of the sun's centre		60 37 9

The above might be performed in one operation by balancing the negatives and affirmatives of the corrections, as in addition of algebra.

4. The next correction necessary to be made is that of the declination for any hour or longitude, from the declination given in the Nautical Almanac: this is performed by turning the given longitude into time, and saying, *As 24 hours is to the daily difference of declination, so is the said time to the difference between the declination at Greenwich and that at the given place.*

Example.

What was the sun's declination at noon on the 5th of May, 1795, in long. 158° (= 10 h. 32 m.)?

Decl. May 5	16° 19' 23"
May 6	16 36 17
Daily diff.	16 54

As 24 h : 16' 54" :: 10 h. 32' : 7' 25".

Then 16° 19' 23" + 7' 25" = 16° 26' 48" = the sun's declination at noon on May 5, 1795, in lon. 158 West.

If

If the longitude had been East, the daily difference must have been found between the declinations of the 4th and 5th of May, and the correction subtracted.

If it had been required to know the declination at any other hour, suppose at 2 o'clock, in long. 158° West, then the time would be two hours later than the above, viz. 12 h. 32'.

5. When the altitude and declination are corrected, the next object is to find the number of degrees from the observer's zenith to the equinoctial; and this is the latitude of the place, to find which the following two Rules must be observed.

1st. *If the zen. dist. and declin. have the same name, that is, if they are both North or both South.*

Rule. Their difference is the latitude. Like the declination, when it is greater than the zenith distance; but unlike, when it is less.

2d. *If the zen. dist. and decl. have contrary names.*

Rule. Their sum is the latitude, and is always like the declination.

Observe. The zenith distance is said to be North or South, according as the sun is North or South of the zenith.

The above Rules are best illustrated by a globe; thus, mark the sun's declination on the brazen meridian, and make the distance from this point to the horizon equal to the given altitude; then will the globe be set to the

N

latitude

latitude of the place, and the distance from the said point to the zenith will also shew the latitude.

Example.

In which the foregoing Rules and Corrections are introduced.

May 5, 1795, in long. 158° West, the meridian altitude of the sun's upper limb was found 17° 47' 25", the sun being South of the zenith, and the observer's eye 24 feet above the water, required the latitude.

Observed alt. of the sun's upper limb	47° 37' 25"
Refraction	— 0' 52"
Dip. of the horizon	— 4 40
Sun's sem. diam.	— 15 54
parallax in alt.	+ 6
	21 20
True alt. of the sun's centre	47 16 15
Zen. dist. <i>South</i>	42 43 55
Sun's decl. at noon at Greenwich	16° 19' 23"
Diff. in decl. for long. 158°	+ 7 25
Sun's true decl. <i>North</i>	16 26 48
The latitude of the place	N. 59 10 43

The following problems are numbered after those of the foregoing Section, and being partly deduced from them general directions only are given with the answers, the rest being left for the exercise of the learner. Here, as well as in many other parts, the methods may be varied, particularly those of the Projection.

PRO-

PROBLEM 14. FIG. 40.

Given the sun's altitude at six o'clock, and the day of the month.*

Required the latitude of the place.

Exam. On the 21st of June, 1795, the sun's altitude at six o'clock in the evening was observed to be $18^{\circ} 9' 55''$, what was the latitude of the place?

Here in the right angled spheric triangle $\gamma \odot A$, right-angled at A , there are given the alt. $A \odot = 18^{\circ} 9' 55''$, and the decl. $\gamma \odot = 23^{\circ} 27' 51''$, to find the latitude $\angle A \gamma \odot$.

Thus are given the hyp. $\gamma \odot$, and the leg $A \odot$, to find the angle opp. the given leg, which may be performed by Art. CLVIII, and the Figure constructed by Art. CXXI, then will the lat. be found $= 51^{\circ} 32'$.

PROBLEM 15. FIG. 40.

Given the sun's azimuth at six o'clock.

Required the latitude of the place.

Exam. June 21, 1795, at six o'clock in the evening, the sun was observed to bear $74^{\circ} 53' 22''$ West from the North point; what was the latitude of the place?

Here in the right angled spheric triangle $Z \odot P$, right angled at P , there are given the azimuth angle $\odot Z P$

$$N \quad \quad \quad = 74^{\circ}$$

* The day of the month being given, the sun's declination is thence known; but the repetition of either is omitted in the following Problems of this Section, as it is always understood that the day is known: the declinations are here taken for noon, without any correction.

$= 74^{\circ} 53' 22''$, and the co-decl. $\odot P = 66^{\circ} 32' 9''$, to find the leg ZP the co-lat.

Thus are given a leg and its opp. angle, to find the other leg, which is performed by Art. CLXIII, and the Figure constructed by Art. CXXVI: thus the co-latitude is found $= 38^{\circ} 28'$.

PROBLEM 15. FIG. 41.

Given the sun's altitude when due East or West.

Required the latitude of the place.

Exam. June 21, 1795, the sun's altitude when due East was observed to be $30^{\circ} 34' 1''$. *Required* the latitude of the place.

Here in the right angled spheric triangle $\nabla A \odot$, right angled at A, there are given the alt. $\nabla \odot = 30^{\circ} 34' 1''$, and the decl. $A \odot = 23^{\circ} 27' 51''$, to find the latitude $\angle A \nabla \odot$.

Thus are given the hyp. $\nabla \odot$ and the leg $A \odot$, to find the angle opp. the given leg, which is performed by Art. CLVIII, and the Figure projected by Art. CXXI.

Hence the angle $A \nabla \odot$ the latitude will be found $= 51^{\circ} 32'$.

PROBLEM 16. FIG. 42.

Given the sun's amplitude.

Required the latitude of the place.

Exam. On the 21st of June, 1795, the sun was observed to rise $39^{\circ} 48' 3''$ from the East point northerly. *Required* the latitude of the place.

Here

Here in the right angled spheric triangle $\gamma \odot A$, right angled at A , there are given the amp. $\gamma \odot = 39^\circ 48' 3''$, and the decl. $\odot A = 23^\circ 27' 51''$, to find the co-lat. $\angle A \gamma \odot$.

Thus are given the hyp. $\gamma \odot$, the leg $A \odot$, to find the angle at γ opposite the given leg: the solution and construction are performed as in the last problem, which will give the co-lat. $\angle A \gamma \odot = 38^\circ 28'$.

PROBLEM 17. FIG. 42.

Given the time of the sun's rising and setting.

Required the latitude of the place.

Exam. On the 21st of June, 1795, the sun was observed to set at 8 h. 12' 28", 13. What was the latitude of the place?

Here the time after fix o'clock, viz. 2 h. 12 m. 28 s. ,13, being turned into degrees $= 33^\circ 7' 2''$, which is the ascensional difference γA . Hence there are given in the right-angled spheric triangle $\gamma \odot A$, right angled at A , the ascensional diff. $\gamma A = 33^\circ 7' 2''$, and the decl. $A \odot = 23^\circ 27' 51''$, to find the co-lat. $\angle A \gamma \odot$.

Thus the two legs are given to find an angle, which is performed by Art. cxix, and the figure may be constructed by Art. cxxx. The angle $A \gamma \odot$ the co-lat. $= 38^\circ 28'$.

PROBLEM 18. FIG. 44.

Given the sun's altitude and the hour.

Required the latitude of the place.

N 3

Exam.

Exam. At 2 h. 55 m. 26 s. on the 21st of June, 1795, the sun's altitude was observed to be $46^{\circ} 20'$. What was the latitude of the place of observation?

Here in the oblique angled spheric triangle $Z \odot P$ there are three things given, viz.

the co-alt. or zen. dist. $Z \odot = 43^{\circ} 40'$,
 the co-decl. or polar dist. $P \odot = 66^{\circ} 32' 9''$,
 and the horary angle $Z P \odot = 43^{\circ} 50' 3'' (= 2 h. 55' 26'')$

To find the side $Z P$ the co-lat.

Thus two sides and an opp. angle are given to find the third side, which is performed by Case 1, page 74 or 87, and the Figure may be projected by Art. cxxxii. Hence the co-lat. $Z P$ is found $= 38^{\circ} 28'$.

PROBLEM 19. FIG. 44.

Given the sun's altitude and azimuth.

Required the latitude of the place.

Exam. June 21, 1795, the sun's altitude was observed to be $46^{\circ} 20'$, when his azimuth was $113^{\circ} 3' 18''$, that is, his bearing from the West was $66^{\circ} 56' 42'' S$. Required the latitude of the place of observation.

Here in the oblique angled spheric triangle $Z P \odot$ there are given three things, viz.

the co alt. or zen. dist. $\odot Z = 43^{\circ} 40'$
 the azimuth angle $\odot Z P = 113^{\circ} 3' 8''$
 and the co decl. or pol. dist $\odot P = 66^{\circ} 32' 9''$.

To find the co-lat. $Z P$.

Thus

Thus two sides and an opp. angle are given to find the third side; the solution is therefore the same as in the last problem.

The latitude may be likewise found from double altitudes of the sun, equal or unequal, examples of which are given in most books of navigation.

The latitude may be found a variety of ways by means of the fixed stars. Thus the altitude of the polar star is nearly the latitude of the place, and half the sum of the greatest and the least altitude of any star near the pole will give the latitude. It is also obtained from the altitude of any star whose declination is known (the hour or azimuth being given) on the same principles that the latitude is found from the altitude of the sun, Prob. 18 and 19. In short, this important knowledge of the latitude may be obtained by a great variety of methods, which afford useful practice in Spheric Astronomy. The student who wishes for further exercise on this subject is referred to those editions of Robertson's Navigation which have been revised and augmented by Mr. Wales.

SECTION X.

UPON THE LONGITUDE.

GENERAL REMARKS UPON THE LONGITUDE, WITH
A COMPARATIVE VIEW OF THE VARIOUS METHODS
OF DETERMINING THIS IMPORTANT QUESTION.

THE great object of Nautical Astronomy is to tell at all times *the place where the ship is*; in other words, to find the latitude and longitude. This knowledge is sometimes obtained by the common reckoning at sea of course and distance made good; but various, unavoidable, and often imperceptible causes, render this kind of computation erroneous; recourse must therefore be had to other methods, and the heavenly bodies, the only visible guides at sea, afford the surest means, even at land, for finding the latitude and longitude.

In the last Section it was shewn, that the latitude is found by the meridian altitude of some celestial object whose declination is known, or from some other things which have a known relation or reference to the meridian circle; but there are no such data for determining the longitude. Had there been any fixed object in the eastern or western hemispheres, its altitude would shew the longitude upon the same principles that the polar star, or any other object in the great circle of the meridian, shews the latitude; but the diurnal rotation of the earth on its axis, and its annual revolution round the sun, cause a continual change in the altitudes of the heavenly bodies in the above direction, and every change of latitude produces a still greater variation in

in those altitudes; hence the longitude can never be found by the height of any celestial object, without a laborious and doubtful calculation.

The difficulty of finding the longitude by any easy and practical method, and the great importance of this Problem to navigation and commerce, have induced different governments to offer immense rewards for the discovery. Such encouragement, united with the powerful incitements of emulation, honor and ambition, have called forth the most extraordinary efforts of genius, and have produced great improvements both in mechanics and astronomy; and notwithstanding the accuracy with which the longitude is now determined, considerable rewards are still held out for the discovery of a more practical method.

It may seem a matter of astonishment that this question, which is probably the most interesting that ever engaged the human attention, is little more than to be able to tell *what o'clock it is elsewhere*; for the longitude is found by the comparison of local or relative time, and as the hour is easily found at the place of observation by altitudes, or otherwise, the only difficulty is to find the time at some other place whose longitude is known. For as the sun in his daily course passes over 360 degrees of longitude in 24 hours, he passes over 15 degrees in one hour, and over any other space in this proportion; and therefore if the difference between the times of any two places be known, and this turned into degrees (p. 144) the difference of longitude is thence found.

Hence if a perfect time-keeper could be constructed it would obviate all difficulty on this subject, and render the

the longitude as simple a problem as the latitude; for such an instrument being set to the time of any place whose longitude is known (suppose to that of Greenwich Observatory, from whence we reckon our longitude) it would preserve this time in all other parts of the world, and by comparing this chronometer with a clock or watch properly regulated for the place of observation, the difference would shew the longitude.

Notwithstanding the great degree of perfection to which time-keepers have been brought, they cannot be supposed such infallible guides to the longitude as the heavenly bodies; the only advantage the former have is that of being most easily consulted, but the prudent mariner will not trust to chronometers alone, though he may use them as helps or checks to his astronomical calculations; for such delicate and complicated pieces of mechanism must be ever more or less liable to be affected by the violence of motion or the vicissitudes of season and climate, and must moreover, like all other productions of human art, be subject to accident, disorder and decay; whereas the heavenly bodies are unchangeable, these only are the unerring time-keepers, which exhibit a true specimen of perpetual motion.

There are various methods of finding the longitude by celestial observations, but all proceed on the same principle or tend to the same object, which is *to tell the time at Greenwich Observatory*; and this compared with the time at the place of observation shews the distance East or West from the above place, and of course shews the longitude.

Now the Greenwich time is found by the help of the Nautical Almanac, which is calculated for the meridian
of

of that place. Here the time is shewn when various celestial appearances take place under the above meridian, and as those appearances are seen nearly at the same moment of absolute time in every part of the world where they are visible, *the difference between the observed time of any phenomenon and the computed time in the almanac will give the longitude.*

Thus, suppose the time when an eclipse of the moon was observed to happen be 12 o'clock at night, but by the almanac it will not take place until 3 o'clock in the morning, it is evident that the longitude of the place of observation is 45 degrees, and that it is West of Greenwich, the time being more advanced in the East than in the West.

By this simple method the lunar eclipses will shew the longitude, but they happen too seldom to answer the general wants of navigation.

The eclipses of Jupiter's moons will also shew the longitude upon the same principles, and these happen sufficiently often, but there are two inconveniences to which they are liable; the first is, that Jupiter is visible only at certain seasons; the second, that his satellites are not visible to the naked eye; and as they must be viewed with a telescope, they can only determine the longitude at land, for the motions of a ship render all telescopic observations impracticable at sea. This is an inconvenience to which perhaps no adequate remedy can be ever applied. Some ingenious devices have been tried to make those observations, but hitherto without effect. Such an invention, next to that of a perfect time-keeper, is perhaps the greatest *desideratum* in navigation.

The

The longitude may be also determined by the solar eclipses, by the sun's declination, by the moon's culminating, or by the occultation of a fixed star. These several methods are explained in most books of navigation, but they are not generally practised, a small error in the observation being liable to produce a considerable error in the longitude.

The most practical method that has been yet adopted for determining this problem is by the lunar observations, which shall be particularly explained and illustrated in the following pages.

OF THE LUNAR OBSERVATIONS.

The method of finding the longitude by taking the angular distance between the moon and the sun, or a star, is the greatest modern improvement in navigation. The idea, however, is not modern, but has never been applied with any success until within the last thirty years. M. de la Lande, in his *Astronomy*, mentions some persons who above two hundred years ago proposed this method, and contended for the honor of the discovery; but its present state of improved and universal practice he very justly ascribes to Dr. Maskelyne, the present Astronomer Royal of England. The discovery of this method, however, seems to claim less honor than its subsequent improvement; it is one of those things which are obvious in theory, but difficult in practice; in short, more easily designed than executed. The most ancient method of finding the longitude was by the lunar eclipses; and that of finding it by the lunar distances is perfectly analogous; the latter, for instance, shews when there is a certain distance between the sun and
moon,

moon, the former when they are at the greatest distance; it is therefore highly probable that this method was thought of at a very early period, but the want of proper documents and apparatus prevented its being put into execution.

It may be observed, that in the most practical methods of finding the longitude by celestial observations the moon is the chief guide or instrument, for the quickness of her motion renders her peculiarly well adapted for measuring small portions of correspondent time: her periodical revolution round the earth is performed in about 27 days, and of course she moves 13° nearly in one day, which is about one minute of a degree in two minutes of time: thus if she be observed near any fixed star at a certain hour, she will be seen at the same hour the next day considerably removed from the star toward the East, and will be observed to rise, set, culminate or pass any fixed object in the heavens about three quarters of an hour later each day than on the preceding one, for as $360^{\circ} : 24 \text{ h.} :: 13^{\circ} : 46'$ nearly.

Now as the moon is seen in the same part of the heavens nearly at the same instant of absolute time from all parts of the earth where she is visible, and as she is continually and sensibly changing her place, it is evident that if two correspondent observers were to note the precise moment of their respective times when she was seen in any particular part of the heavens, the difference between those times would shew the difference of longitude.

In every method of finding the longitude by the moon, the first object is to be able to ascertain the part
of

of the heavens where she is: this is easily seen at the time of her eclipses, or at the occultation of a fixed star, and these were naturally the first methods resorted to, but they occur too seldom for general use; the moon's place, however, may be marked when she is visible with equal precision, by taking her distance from some fixed object in the zodiac. Thus she may be compared to a traveller, whose situation is most simply described when at some known place, but his situation may also be ascertained on any part of the road by measuring his distance from a known place.

It has been before observed, that if two persons under different meridians were to mark the moon's place, and also their relative time of observation, they might thence tell their difference of longitude; but as such could not communicate and compare notes sufficiently soon for nautical purposes, and even admitting the possibility of this, it were necessary that the longitude of one should be known in order to determine that of the other. Now the Nautical Almanac is calculated to supply all these wants: here the various phenomena of the heavenly bodies are computed. This admirable work may be considered a perpetual observer, who communicates universally and instantaneously the celestial appearances which may be useful in navigation, as they take place at Greenwich Observatory. Thus the distances are given between the moon and sun and certain remarkable stars in the zodiac for every three hours, and any intermediate distance, or time, may be thence found by the rule of proportion, which will be shewn in its proper place.

Hence it is obvious, that if under any meridian a lunar distance be observed, the difference between the
time

time of observation and the time in the Almanac when the same distance was to take place at Greenwich will shew the longitude. For example, if the observed distance between the sun and moon be 50° at eight o'clock, but by the Almanac the same distance of 50° will take place at Greenwich at six, it is evident that the difference between the observed and computed time is two hours, and therefore the longitude is 30° ; and it is also clear that the longitude is East; the time being latest at the place of observation.

A method so apparently simple must have been long since adopted, but two difficulties occurred, which were not removed until the present century; one was the want of proper instruments, which want has been happily supplied by the invention and subsequent improvements of Hadley's quadrant: the other was the want of correct Lunar Tables; for the moon, though so near and conspicuous to the earth, has always perplexed astronomers more than any other planet. The various inequalities of her motions were never properly understood, until Sir Isaac Newton discovered the physical laws which governed them, and from his theory the late Professor Mayer of Gottengen formed Lunar Tables, which are found sufficiently correct, and from which those in the Nautical Almanac of the lunar distances are calculated.

Here two difficulties being obviated, a third seemed to occur; and though this is removed by the application of the Requisite Tables and Nautical Almanac, published by order of the Commissioners of Longitude, and under the inspection of the Astronomer Royal, yet the operation is still more tedious than might be wished, nor

is it possible to make it much shorter. This arises from the *observed distances* between the heavenly bodies not being the *true distances*; for as the altitudes of those bodies are more or less affected by refraction and parallax, and though these effects only operate in a vertical direction, yet that which changes the altitudes of two bodies must also change their distance asunder. This is evident from the consideration, that the altitude of a celestial object is an arc of an azimuth circle intercepted between the object and the horizon. Now as all azimuth circles incline gradually to each other from the horizon to the zenith, where they meet, it is plain that the more two bodies are apparently raised, the less will be their apparent distance asunder; and on the contrary, the more they are apparently depressed, the greater will be their apparent distance.

It was observed before, that the heavenly bodies are raised by refraction and depressed by parallax, and that these effects are greatest in the horizon, but gradually diminish to the zenith, where they become nothing. Now all celestial objects, except the moon, are more affected by refraction than by parallax, and therefore appear above their true places; but the moon is always, excepting in the zenith, seen below her true place, being more affected by parallax than refraction, owing to her proximity to the earth: the fixed stars have no sensible parallax.

These effects of parallax and refraction, though counteracting each other, scarcely ever do it so equally as to render all correction unnecessary. Sometimes the apparent distance of the moon from the sun, or a star, is nearly a whole degree more or less than the true distance.

In

The principal cause of so great a difference is the moon's parallax; for this body, which is the chief guide in finding the longitude, is also the great cause of error in the distances, and is of course the principal object of correction.

In making a lunar observation four persons are generally employed, one of whom takes the distance, two the altitudes, and the fourth notes the time: these things should be performed at the same instant; and if the observation be repeated several times, and a mean taken, the work is likely to be the more correct; and every care is here necessary, for the smallest error in this part of the operation, particularly in taking the distance, will pervade the subsequent parts of the work, and will of course produce a wrong solution. The manner of adjusting the instruments and of making the observation is best taught by practice; but those who wish for written instructions on the subject are referred to the British Mariner's Guide, or to Mr. Mackay's book upon the Longitude.

OF CORRECTING THE ALTITUDES OF THE OBSERVED OBJECTS.

When a lunar observation is made, the first thing to be done is to clear the altitudes from semidiameter, dip, refraction and parallax, and this is performed upon the same principles as in Page 175 for finding the latitude.

In correcting the moon's altitude, her parallax must be previously calculated; which is done by saying, *As radius is to the sine of her zenith distance, so is the sine of her horizontal parallax (as given in the Nautical Almanac) to the sine of her parallax in altitude.*

O

In

In correcting the moon's altitude an allowance must be made for the augmentation of her semidiameter, which gradually takes place from the horizon to the zenith. This increase is given, in the IVth of the Requisite Tables, for every five degrees of altitude, which correction is to be added to her horizontal semidiameter given in the Nautical Almanac.

The augmentation of the moon's semidiameter is caused by her being nearer to the spectator in the zenith than in the horizon by a semidiameter of the earth. For the magnitude of a body is in the inverse ratio of its distance from the observer; and as the earth's semidiameter bears a very sensible proportion to the moon's distance, she is seen under the greatest angle in the zenith, which angle gradually diminishes to the horizon. This seems contrary to appearance, but it is confirmed by observation.

There are some other corrections of the altitudes which may be necessary in cases of peculiar nicety, but are seldom noticed at sea. These are,—an allowance for the contraction of the vertical semidiameters of the sun and moon by refraction:—a correction of the moon's parallax, supposing, according to Sir Isaac Newton's hypothesis that the earth is a spheroid:—a correction for the refraction according to the actual state of the atmosphere as shewn by a thermometer and barometer, and not according to the mean astronomical refraction which is commonly used. These corrections, though perhaps necessary toward the perfection of the longitude, being very small, and frequently counteracting each other, are generally considered of little consequence in nautical practice, where greater errors are unavoidable.

From

From the corrected Altitudes to find the true Distance.

It is easy to conceive, that by a lunar observation the three sides of a spheric triangle are measured in the heavens, which are the apparent co-altitudes of the observed bodies and their apparent distance asunder.

The co-altitudes or zenith distances being corrected, the question is to find the true distance between the observed bodies; but here there are only two things given, and therefore it cannot be done until the angle at the zenith is known, which is determined by Art. CLXXXII, from the three given sides of the triangle.

As the effects of parallax, refraction, &c. operate only in a vertical direction, it is evident that the corrections of the zenith distances or containing sides will not change the included angle at the zenith; and therefore three things are now known, namely, the corrected zenith distances and the included angle, from whence the other side is determined by Art. CXC, and this side is the true distance sought. If the observed bodies be both in the same azimuth circle, the difference between the true altitudes will give the true distance, and then no triangle is formed:—but these things will be more clearly understood by projecting a lunar observation upon stereographic principles, and applying the corrections as in the following Examples.

In the subsequent pages the altitudes are assumed as cleared of dip and semidiameter, these operations being deemed too simple to require any repetition.

To project a Lunar Observation stereographically, and thence to estimate the true Distance.

EXAMPLE I. FIG. 47.

Given the apparent altitude of the moon's centre $22^{\circ} 15'$; that of the sun $21^{\circ} 35'$, the apparent distance between their centres $119^{\circ} 20' 34''$, and the moon's horizontal parallax $58'$; to project the figure and measure the true distance.

The question here is, first to project a spheric triangle ZSM from the three sides given, namely, the co-altitudes or zenith distances of the sun and moon, and their apparent distance asunder; secondly, to apply the corrections of the altitudes for parallax and refraction; and lastly, to draw a great circle through the places of the true altitudes, and measure the intercepted arc, which will shew the true distance nearly.

Here the side SM is given $= 119^{\circ} 20' 34''$, the side SZ (the sun's apparent zenith distance) $= 68^{\circ} 25'$, and the side MZ (the moon's apparent zenith distance) $= 67^{\circ} 45'$; to project the triangle, which is performed by Art. cxxxvi, thus:

1. With the sweep of 60 from the scale of chords describe the primitive circle HZRN, and draw ZN and HR at right angles to each other.

Then Z represents the zenith and HR the horizon.

2. As the moon's altitude requires the greatest correction, let this be put on the primitive, that is, lay off $22^{\circ} 15'$ from the scale of chords from R to M; or what is the same thing, lay $67^{\circ} 45'$ from Z to M, then is M the apparent place of the moon.

3. Draw

3. Draw the diameters MA and BC at right angles to each other.

4. Describe the small circle hr parallel to HR with the tan. $68^{\circ} 25'$ (the sun's zenith distance) cxiv: thus, on the line NZ continued lay off from the centre of the primitive the secant $68^{\circ} 25'$, which will give the centre of hr; then with the tangent $68^{\circ} 25'$ and one foot in this centre describe hr.

5. In the same manner describe the small circle bc parallel to BC with the tangent $60^{\circ} 39' 26''$ (the supplement of the apparent distance). Where bc and hr intersect in S is the apparent place of the sun.

6. Through the three points N, S, Z, draw the great circle NSZ, and through M, S, A, draw the great circle MSA. (xcv); then is MS the apparent distance between the sun and moon, and ZSM the triangle required,

To apply the Corrections.

7. Let the moon's parallax in altitude be found by logarithms, or by the line of fines on the scale, and let this be lessened by her refraction (Tab. I.) and lay the difference off upon the primitive from M to m.

This correction may be assumed something less than a degree on the scale; and as the moon appears below her true place, she is to be here raised, that is, the correction is to be added to her altitude, or subtracted from her zenith distance.

8. As the sun appears above his true place, his refraction lessened by his parallax (I and III Req. Tab.) must be subtracted from his altitude, or added to his ze-

nith distance; and as this is a very small correction, especially when his altitude is great, it may be assumed in the projection as near S as possible, so as to make a visible distinction between the points S and s; then s represents the sun's true place.

9. Through the points m, and s, draw the arc of a great circle (xcvi. 2), then will sm represent the true distance, which may be measured by Art. ciii, and will be found about half a degree less than the apparent distance.

10. The true distance sm may be here estimated, by considering that the apparent distance SM is nearly double the sun's zenith distance SZ; and therefore as the chief correction is applied from M toward Z, the arc SM will be lessened at every such change about one half as much as the arc ZM.

11. Hence it is evident, that when the observed distance is greater than the star's co-altitude, the correction is subtractive, that is, the true distance is less than the apparent: an exception to this sometimes takes place when the star's altitude is under ten degrees, but when the distance is above 90 degrees, it may be taken for a general rule that the correction is subtractive; and this is mostly in proportion to the altitude of the sun or star.

EXAMPLE II. FIG. 48.

Given the moon's apparent altitude $15^{\circ} 21'$, that of a star $78^{\circ} 18'$, their apparent distance $65^{\circ} 27' 30''$, and the moon's horizontal parallax $60' 25''$; to project the figure, &c.

1. Let the primitive be drawn with diameters at right angles, as in the last Example, and lay off the moon's apparent altitude RM.

2. Draw

2. Draw MA and BC at right angles ; and parallel to HR and BC draw the small circles hr and bc ; the first with the tan. $11^{\circ} 42'$ (the sun's apparent zenith distance) and the second with the tan. $65^{\circ} 27' 30''$ (the apparent distance) cxiv.

3. Where these small circles hr and bc intersect in S is the sun's apparent place, and therefore through the points M, S, A, and Z, S, N, draw great circles (xcv) ; then is SM the apparent distance, and ZSM the triangle required.

To apply the Correction.

4. Here the moon's altitude being small, her parallax may be assumed nearly a degree, and let this be laid off on the primitive from M to m, which will give m the moon's true place; and the star's altitude being great its refraction is so very small, that s its true place may be assumed as near S as possible, and let the arc sm be drawn (xcvi. 2) which is the true distance, and being measured by Art. cxii, will be found nearly a whole degree less than the apparent distance SM. For the moon and star being here nearly in the same azimuth circle, the moon's correction which raises, and the star's which depresses, make those objects as it were approach toward each other, and this nearly the sum of their corrections. If the observed bodies had been both exactly in the same azimuth circle ZR, the sum of the corrections of their altitudes would be the correction of their distance. Thus if the moon were the lower body, and the correction of its altitude $30'$, and that of the star's altitude $5'$, it is evident that the true distance would be 35 minutes less than the apparent; and on the contrary, if the moon be the higher body, and the corrections of

their altitudes be $5'$ and $30'$ respectively, it is clear that the true distance will be 35 minutes more than the apparent.

From these considerations it is manifest, that when the moon is the lower body, the correction is always subtractive, that is, the true distance is less than the apparent: but the converse of this only holds in a certain degree, for when the moon is the higher body, the true distance is not always greater than the apparent, though it can never be so but in this situation.

When the sine of the moon's altitude is to that of the star's as radius is to the co-sine of the observed distance, then there will be little or no correction necessary; but when the moon's altitude is greater than the star's beyond this proportion, the correction is additive except when the distance is above 90 degrees.

Although the correction additive does not occur so frequently as the subtractive one, nor is it in general so great, yet in some particular cases it is the greatest: when the star's altitude is about 5° and the moon's 25° in the same azimuth circle, the correction will be sometimes above a degree additive; but the correction subtractive is never so much as this.

To solve a Lunar Observation by a new and simple Projection.

The calculation of this important problem being more prolix and operose than any other in Nautical Astronomy, different methods have been devised for obtaining a rough solution by Projection or by a graphic operation. The first performance of this description

was

was the *Cassie de Reduction* of la Caille: other ingenious methods have been since contrived, but the most approved performance of the graphic kind is that lately executed by Mr. Margetts; a work of great labour, ingenuity and correctness.

The following method is different from any hitherto devised, both in form and simplicity of operation, being immediately performed by drawing four right lines; and though it cannot be supposed that so complicated a problem can be solved with perfect accuracy by such a simple projection, yet it will be found to give the solution with surprising exactness, especially when neither of the observed bodies is very near the horizon or the zenith. And such extreme altitudes should not be chosen when others can be had, being most liable to error, by whatever method the work is performed.

The scales which are to be used in this Projection are the line of chords and its corresponding line of sines; the altitudes are laid off from the latter and the distance from the former, and the correction is also measured by the line of chords, every degree of which is reckoned only a minute of correction: this circumstance of so large a scale is of great advantage.

R U L E.

Having the apparent altitudes and distance given, and the moon's horizontal parallax as before, draw a line at pleasure, which call the *Lunar Line*.

Draw another line to make an angle with the lunar line equal to the apparent distance, which may be called the *Solar-Line*.

From

From the angular point lay off the sine of the moon's and of the sun's altitudes upon their respective lines, and mark the points to which they extend.

Through each of these points draw a line perpendicular to the lunar and solar lines, and mark where these two perpendicular lines intersect each other.

The nearest distance from the lunar line to the point of intersection will give the first correction, reckoning each degree of the scale a minute of correction, and let this be called the *Line of Correction*.

The correction is always subtractive, when the line of correction is at the solar side of the lunar line, but when it is at the other side the correction is additive.

From the Line of Correction and the Moon's horizontal Parallax, to find the second or parallactic Correction.

If the correction be subtractive, divide it by 7, and say, as 9' (the difference between 53' and 62', the greatest and least horizontal parallax) is to the quotient, so is the difference between the given horizontal parallax and 62' (the greatest parallax) to a fourth number; which subtracted from the first correction will give the true correction.

If the correction be additive, divide it by 6, and say, as 9' is to this quotient, so is the difference between 53' (the least horizontal parallax) and the given horizontal parallax to a fourth number; which added to the first correction will give the true correction.

EXAMPLE I. FIG. 49.

Given the moon's apparent alt. $22^{\circ} 15'$
 the sun's or star's apparent alt. $21^{\circ} 35'$
 the apparent distance $119^{\circ} 20' 34''$
 the moon's horizontal parallax $0^{\circ} 58'$

To find the correction and true distance.

Draw the lunar line AB at pleasure, and draw the line AC to make an angle with the former equal to the observed distance.

From the point A lay off the sine of the moon's altitude to $A\Delta$, and that of the star to $A*$.

Through the points Δ and $*$ draw perpendicular lines, which will intersect in D ; then is $D\Delta$ the first correction, which, measured on the line of chords, and calling each degree on the chords a minute, will be found $= 36'$, and being on the solar or star's side of the lunar line it is subtractive.

To find the Corrections for Parallax.

$$\frac{36}{7} = 5\frac{1}{7}: \text{then } 9 : 5\frac{1}{7} :: 4 : 2\frac{1}{3} \text{ nearly,}$$

$$\text{and } 36 - 2\frac{1}{3} = 33\frac{2}{3} \text{ the correction.}$$

App. dist. $119^{\circ} 20' 34''$

Correction $0^{\circ} 33' 40''$

118 46 54 true distance, which is $1''$ less than the result by calculation, see page 208.

E X.

E X A M P L E II. FIG. 50.

Given Moon's apparent altitude	15° 21'
Star's ditto	78 18
Apparent dist.	65 27 30"
Moon's horizontal parallax	1 0 25

To find the correction and true distance.

Let ABC be described to make the angle at A = $65^{\circ} 27' 30''$, the apparent distance, and lay off from the line of fines A D and A * equal to the moon's and star's altitudes respectively.

Through D and * draw perpendicular lines which will meet in D, then is D D = 56' the first correction, and is subtractive, being on the side of the star.

To find the Correction for Parallax.

$$\frac{56}{7} = 8 : \text{then } 9 : 8 :: 1\frac{1}{4} (62 - 60\frac{1}{4}) : 1\frac{1}{2} \text{ nearly.}$$

Then $56 - 1\frac{1}{2} = 54\ 30$ the correction.

Apparent dist.	65 27 30
Correction	0 54 30

64 33 0 the true distance, being 2" less than the result by calculation, see the last plate.

E X A M P L E III. FIG. 51.

Given the Moon's apparent altitude	= 55° 56'
the Star's apparent altitude	19 18
Apparent distance	50 8 41"
Moon's horizontal parallax	1 0 15

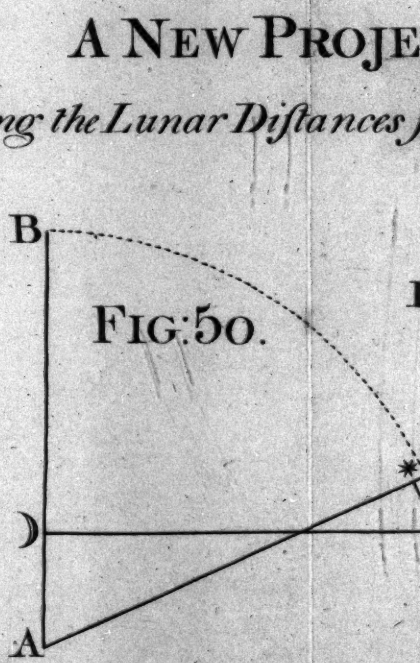
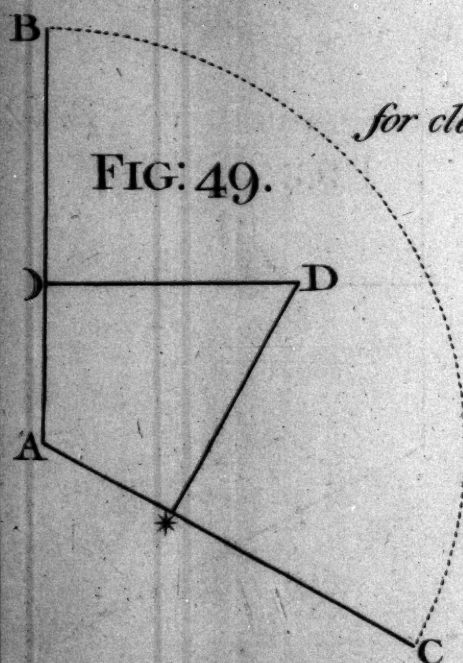
To find the correction and true distance.

Draw ABC as before, making the angle at A = $50^{\circ} 8' 41''$, the apparent distance.

0°
earch
ations

A NEW METHOD of working the Lunar Observations.				
GIVEN	½ Apparent Altitude.....	15. 21. 0	To find the True distance	
	*½ Apparent Altitude.....	78. 18. 0		
	Apparent Distance.....	65. 27. 30		
	½ Horizontal Parallax.....	1. 0. 25		
½ Horizontal Parallax.....	1. 0. 25	S	8.2448605	
½ Apparent Zen. dist.....	74. 39. 0	S	0.0842242	
½ Parallax in Altitude.....	0. 58. 16	S	8.2200847	
½ Refraction.....	3. 26			
½ Correction.....	0. 54. 50	True Alt.	16. 15. 50	
*½ Corr ⁿ or Refrac ⁿ	0. 0. 12	True Alt.	78. 17. 48	
Apparent Distance.....	65. 27. 30			
½ Apparent Zen. dist.....	74. 39.	Co-Ar. S	0.0157758	
*½ Apparent Zen. dist.....	11. 42.	Co-Ar. S	0.6929593	
Sum.....	151. 48. 30			
½ Sum.....	75. 54. 15			
1 st Remainder.....	1. 15. 15	S	8.3401970	
2 nd Remainder.....	64. 12. 15	S	0.9544116	
½ True Zen. dist.....	73. 44. 10	S	0.0822631	
*½ True Zen. dist.....	11. 42. 12	S	0.3071627	
		Sum	38.2927704	
		½ Sum	19.1463852	
½ diff ⁿ True Zen. dist ⁿ	31. 0. 59	S	9.7120460	
Tangent of Arc.....	15. 12. 31		0.4343392	
Same Arc.....	15. 12. 31	S	0.4188540	
½ True Distance.....	32. 16. 31	S	0.7275303	
TRUE DISTANCE	64. 33. 2			

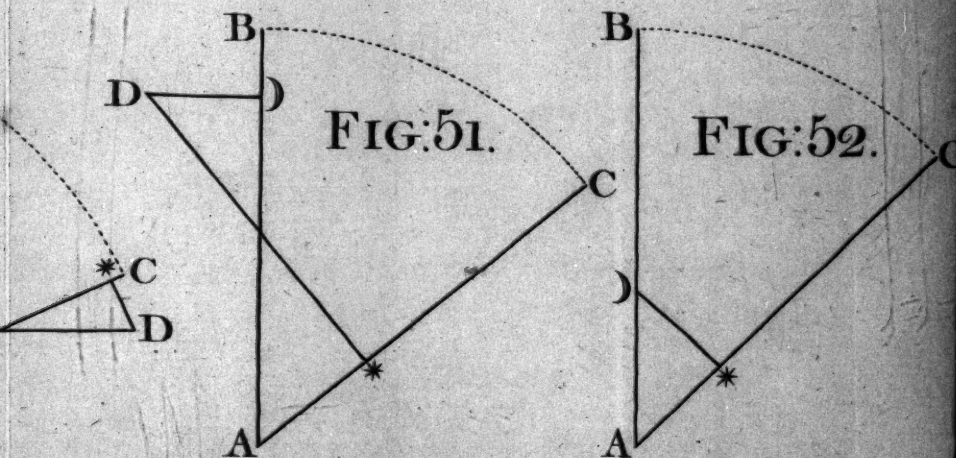
Engraved for Kelly's Spherical Trigonometry
by J. Johnson, St. Pauls Ch. Yard. &c



A FORMULA			
for working the Lunar Observations.			
GIVEN	{		To find the True distance
	1/2 Apparent Altitude		
	* 1/2 Apparent Altitude		
	Apparent Distance		
	{		
	1/2 Horizontal Parallax		
1/2 Horizontal Parallax		S	
1/2 Apparent Zen. dist.		S	
1/2 Parallax in Altitude		S	
1/2 Refraction			
1/2 Correction		True Alt.	
* 1/2 Corr. ⁿ or Refrac. ⁿ		True Alt.	
Apparent Distance			
1/2 Apparent Zen. dist.		Co Ar. S	
* 1/2 Apparent Zen. dist.		Co Ar. S	
Sum			
1/2 Sum			
1 st Remainder		S	
2 ^d Remainder		S	
1/2 True Zen. dist.		S	
* 1/2 True Zen. dist.		S	
		Sum	
		1/2 Sum	
1/2 diff. ^{ce} True Zen. dist. ⁿ		S	
Tangent of Arc			
Same Arc		S	
1/2 True Distance		S	
TRUE DISTANCE			
Sold separately on Cards by W & S Jones, Optic ⁿ , Holborn			

PROJECTION

Distances from Parallax & Refraction.



Lay off $A D$ and $A * =$ the fines of the altitudes respectively.

Through $*$ and D draw perpendicular lines which will intersect in D , then is $D D = 15\frac{1}{2}$ the first correction, and being on the contrary side of the star it is additive.

To find the Correction for Parallax.

$$\frac{15\frac{1}{2}}{6} = 2\frac{1}{2} \text{ nearly, and } 60\frac{1}{4} - 53 = 7\frac{1}{4}.$$

Then $9 : 7\frac{1}{4} :: 2\frac{1}{2} : 2$ nearly.

$15\frac{1}{2}$ first correction.

Correction + $17\frac{1}{2}$

App. dist. $50 \quad 8 \quad 41$

$50 \quad 26 \quad 26$ true dist.

E X A M P L E. IV. FIG. 52.

Given Moon's apparent altitude = $21^{\circ} 26'$

Star's apparent altitude $15 \quad 54$

Apparent distance $45 \quad 40 \quad 14$

Moon's horizontal parallax $56 \quad 30$

To find the correction and true distance.

Describe ABC as before, making the angle at $A =$ the apparent distance.

And lay off $A D$ and $A * =$ the fines of the altitudes respectively.

Through D and $*$ draw perpendicular lines, which will meet in or near D , and therefore it appears that there is little or no correction to be made, and

By

By calculation the true distance is found only 7" less than the apparent.

This projection shews, that when the sine of the angle $A D *$ (the complement of the distance) is to the sine $A *$ (the star's altitude) as radius (or $D * A$) is to $D A$ (the moon's altitude) then the correction must be small.

This rule, which was given in page 200, may be found very useful as observations made when the bodies are thus situated will require no correction, unless perfect accuracy be sought, and this can be attained by calculation only.

To find the true Lunar Distance by Calculation.

It was before explained, that a lunar observation determines the three sides of a spheric triangle in the heavens, which are the co-altitudes of the observed bodies and their distance asunder, and that the business of calculation is first to find the true altitudes, next the angle at the zenith, and lastly from the two corrected sides and this included angle to find the third corrected side, which is the true distance.

This operation depends upon the solution of two spheric triangles, in the first of which three sides are given to find an angle, and in the second two sides and an included angle to find the third side. The first result may be obtained by Art. CLXXXII, and the second by Art. CXC, and by observing the similarity between the conclusion of the former Article and the beginning of the latter, the reason of the following contractions will appear evident.

As this method may be learned by persons who have not gone regularly through those Articles, the rule is here repeated

repeated at full length with two Examples, the last of which is given upon the plate with the formula.

In making trials of the different methods of solving a lunar observation an impossible triangle should never be chosen; that is, in assuming the altitudes and distance, no one side of the triangle formed in the heavens should be greater than the other two. This caution against so obvious a mistake might seem unnecessary, had not overights of the kind been published in works of the first reputation.

R U L E

For working the Lunar Observations.

1. To the sine of the moon's horizontal parallax add the sine of her app. zen. distance, the sum, rejecting 10 in the index, gives the sine of her parallax in altitude, and this lessened by her refraction (I Req. Table) will give her correction, which subtracted from her apparent zenith distance, gives her true zenith distance.

2. From the sun's refraction subtract his parallax (I and III Req. Tables) and add the remainder to his apparent zenith distance, which will give the true zenith distance.

3. Add together the apparent distance and the apparent zenith distances, and subtract each of the two latter from the half sum, noting the remainders.

4. Add together the sines of these two remainders, the co-arcs of the sines of the apparent zenith distances, and the sines of the true zenith distances.

5. From half the sum of these six logarithms subtract the sine of half the true zenith distances, and the remainder is the tangent of an arc.

6. Subtract the sine of this arc from the said half sum, and the remainder is the sine of half the true distance.

E X.

EXAMPLE.

Given Moon's apparent altitude $22^{\circ} 15'$
 Sun's apparent altitude $21 \ 35$
 apparent distance $119 \ 20 \ 34''$
 Moon's horizontal parallax 58

To find the true distance.

Moon's horizontal parallax	$0^{\circ} 58' 0''$	S.	8.2271953
Moon's apparent zenith dist.	$67 \ 45 \ 0$	S.	9.9663954
Moon's parallax in alt.	$0 \ 53 \ 41$	S.	8.1935907
Moon's refraction	$- \ 0 \ 2 \ 19$		
Moon's correction	$- \ 0 \ 51 \ 22$	true alt.	$23 \ 6 \ 22$
Sun's correction	$- \ 0 \ 2 \ 16$	true alt.	$21 \ 32 \ 44$

Apparent dist.	$119 \ 20 \ 34$		
Moon's apparent zen. dist.	$67 \ 45 \ 0$	Co-ar. S.	0.0336046
Sun's apparent zen. dist.	$68 \ 25 \ 0$	Co-ar. S.	0.0315714

$2)255 \ 30 \ 34$

$\frac{1}{2}$ sum	$127 \ 45 \ 17$		
First remainder	$60 \ 0 \ 17$	S.	9.9375513
Second remainder	$59 \ 20 \ 17$	S.	9.9345950
Moon's true zenith dist.	$66 \ 53 \ 38$	S.	9.9636838
Sun's true zenith distances	$68 \ 27 \ 16$	S.	9.9685417

$2)39.8695478$

$\frac{1}{2}$ diff. of true zenith dist.	$0 \ 46 \ 49$	S.	19.9347739
			<u>8.1341132</u>

Tangent of the arc	$89 \ 5 \ 36$		11.8006607
same arc	$89 \ 5 \ 36$	S.	9.9999456

$\frac{1}{2}$ true dist.	$- \ 59 \ 23 \ 27 \frac{1}{2}$	S.	9.9348283
	2		

True distance $118 \ 46 \ 55$

repeated at full length with two Examples, the last of which is given upon the plate with the formula.

In making trials of the different methods of solving a lunar observation an impossible triangle should never be chosen; that is, in assuming the altitudes and distance, no one side of the triangle formed in the heavens should be greater than the other two. This caution against so obvious a mistake might seem unnecessary, had not oversights of the kind been published in works of the first reputation.

R U L E

For working the Lunar Observations.

1. To the sine of the moon's horizontal parallax add the sine of her app. zen. distance, the sum, rejecting 10 in the index, gives the sine of her parallax in altitude, and this lessened by her refraction (I. Req. Table) will give her correction, which subtracted from her apparent zenith distance, gives her true zenith distance.

2. From the sun's refraction subtract his parallax (I and III Req. Tables) and add the remainder to his apparent zenith distance, which will give the true zenith distance.

3. Add together the apparent distance and the apparent zenith distances, and subtract each of the two latter from the half sum, noting the remainders.

4. Add together the sines of these two remainders, the co-arcs of the sines of the apparent zenith distances, and the sines of the true zenith distances.

5. From half the sum of these six logarithms subtract the sine of half the true zenith distances, and the remainder is the tangent of an arc.

6. Subtract the sine of this arc from the said half sum, and the remainder is the sine of half the true distance.

E X-

E X A M P L E,

Given Moon's apparent altitude	22° 15'
Sun's apparent altitude	21 35
apparent distance	119 20 34"
Moon's horizontal parallax	58

To find the true distance.

Moon's horizontal parallax	0° 58' 0"	S.	8.2271335
Moon's apparent zenith dist.	67 45 0	S.	9.9663954
Moon's parallax in alt.	0 53 41	S.	8.1935289
Moon's refraction	0 2 19		
Moon's correction	0 51 22	true alt.	23° 6' 22"
Sun's correction	0 2 16	true alt.	21 32 44
Apparent distance	119 20 34		
Moon's apparent zen. dist.	67 45 0	Co-ar. S.	0.0336046
Sun's apparent zen. dist.	68 25 0	Co-ar. S.	0.0315714

2) 255 30 34

$\frac{1}{2}$ sum	127 45 17		
First remainder	60 0 17	S.	9.9375513
Second remainder	59 20 17	S.	9.9345959
Moon's true zenith dist.	66 53 38	S.	9.9636838
Sun's true zenith distances	68 27 16	S.	9.9685417

2) 39.8695478

$\frac{1}{2}$ diff. of true zenith distances	0 46 49	19.9347739
		S. 8.1341132

Tangent of the arc	89 5 36	11.8006607
same arc	89 5 36	S. 9.9999456

$\frac{1}{2}$ true distance	59 23 24	S. 9.9348283
	2	

Truedistance - - 118 46 48

From the true Lunar Distance and the Time to find the Longitude.

R U L E.

Observe in the Nautical Almanac at what time on the given day (or on the preceding or following day) the given distance takes place at Greenwich.

The

The difference between the Greenwich time and the time at the place of observation, converted into degrees, shews the longitude.

The lunar distances in the Nautical Almanac being calculated only for every three hours, the given distance almost always falls between two of these, and therefore the corresponding time must be found by proportional parts, thus:

Find the difference between the two computed distances, and also the difference between the first of them and the given distance, and then say,

As the difference between the computed distances

Is to 3 hours,

So is the difference between the given distance and the first computed one

To the proportional time,

which added to the first or preceding time, shews when the given distance takes place at Greenwich.

EXAMPLE.

On the 7th of June, 1795, at 18h. 20m. 10s. the true distance between the sun and moon was found to be $118^{\circ} 46' 48''$, required the longitude?

In page 68 of the Nautical Almanac, the given distance on the above day will be found to fall between the computed distances for midnight, and XV hours, viz. between $119^{\circ} 27' 4''$ and $117^{\circ} 56' 14''$, the difference between these two is $1^{\circ} 30' 50''$, and between the first of these and the given distance $40' 16''$; then as $1^{\circ} 30' 50''$: 3h. :: $40' 16''$: 1h. 19m. 47s. which being added to 12h. gives 13h. 19m. 47s. the time when the given distance took place at Greenwich, and this subtracted from 18h. 20m. 10s. the time at the place of observation, leaves 5h. 0m. 23s. which converted into degrees (page 144) = $75^{\circ} 5' 45''$ the longitude, and this must be East, the

time being more advanced at the place of observation than at Greenwich.

The most concise and practical method of performing this operation is by proportional logarithms*, thus:

Subtract the proportional log. of the difference between the two computed distances, from the proportional log. of the difference between the given distance and first computed one.

The remainder will be the prop. log. of the excess of time, which is to be added to the preceding time as before. Thus:

Given the dist. $118^{\circ} 46' 48''$

Dist. at midnight 119 27 4 Diff. $0^{\circ} 40' 16''$ P. L. 6503

Dist. at XV h. 117 56 14 Diff. 1 30 50 P. L. 9970

Excess 1 h. 19 m. 47 s. P. L. 3533

* Proportional logarithms are calculated from the common logarithms by subtracting the logarithm of the given number from the logarithm of 3.

Thus to find the prop. log. of $40' 16''$.

Log. of 3 = 47712
 $40' 16''$ reduced to the decimal of a degree } 9.82678
 = .6711, the log. of which is

Prop. log. of $40' 16''$ = 65034

The application of proportional logarithms may be easily understood by performing the problem by common logarithms, and then observing how the logarithm of 3 affects the others with which it is connected. Thus:

As $1^{\circ} 30' 50'' = 1.51388 = \text{Log. } 1800914 = a$

Is to 3 hours = Log. 4771212 = x

So is $40' 16'' = .6711 = \text{Log. } 9.8267878 = b$

$3039090 = x + b$

$1800914 = a$

To 1 h. 19 m. 47 s. = 1.3298 = Log. 1238176 = c = x + b - a = c

Now by the construction of proportional logarithms, it is evident that

x - a = prop. log. a

x - b = prop. log. b

x - c = prop. log. c.

And by the application of proportional logs. $x - b - x - a = x - c$, and by subtraction of algebra and transposition this equation will be $x + b - a = c$, which exactly corresponds with the above result by common logarithms.

This Note has been the longer dwelt upon, as it is believed the subject was not explained before.

F I N I S



